Commun Nonlinear Sci Numer Simulat 59 (2018) 306–318

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Interpreting Popov criteria in Luré systems with complex scaling stability analysis

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ARTICLE INFO

Article history:
Received 13 June 2017
Revised 29 October 2017
Accepted 23 November 2017
Available online 2 December 2017

Keywords:
Absolute stability
Positive realness
Luré problem
Complex scaling

ABSTRACT

The paper presents a novel frequency-domain interpretation of Popov criteria for absolute stability in Luré systems by means of what we call complex scaling stability analysis. The complex scaling technique is developed for exponential/asymptotic stability in LTI feedback systems, which dispenses open-loop poles distribution, contour/locus orientation and prior frequency sweeping. Exploiting the technique for alternatively revealing positive realness of transfer functions, re-interpreting Popov criteria is explicated. More specifically, the suggested frequency-domain stability conditions are conformable both in scalar and multivariable cases, and can be implemented either graphically with locus plotting or numerically without; in particular, the latter is suitable as a design tool with auxiliary parameter freedom. The interpretation also reveals further frequency-domain facts about Luré systems. Numerical examples are included to illustrate the main results.

1. Introduction

Circle- and Popov-type criteria have been developed for coping with the Luré problem in classes of linear dynamic systems subject to sector nonlinearities [1–3]. Circle criteria [4–6] follow from the Kalman–Yakubovich–Popov lemma, or the positive real lemma, which provide ‘quadratic’ Lyapuonv function candidates for us to verify globally exponential/asymptotic stability against sector nonlinearities, or absolute stability, by connecting transfer function positive realness with input/output passivity. Passivity is a concept related to both internal and external stabilities. Also with the positivity and passivity theory, Popov criteria [7–9] guarantee existence of ‘quadratic plus integral of nonlinearity’ Lyapunov function candidates for us to cope with a class of Luré systems also in term of absolute stability. Numerous criteria have been claimed in terms of circle- and Popov-type conditions that are graphically implementable (generally in scalar cases [10]), whereas there are ones [11–15] that can be employed algebraically and geometrically; in particular, their LMI interpretation renders us a powerful technique [16,17], just mentioning that LMIs are numerically tractable and thus suitable as a design tool.

As well known, both types of criteria can be utilized for stability analysis and stabilization under a variety of Luré configurations [18–21]. For example, the study in [22] is extended for fuzzy control, while those in [23–25] are devoted to time-delayed systems. Stability issues are considered also in discrete-time [26,27] and switched Luré systems [28,29]. Recently, some interesting results are reported about circle-like stability conditions in descriptor systems subject to sector nonlinearities [16,17]. Sequential circle-like conditions are developed for synchronous generator stabilization in [30]. When discontinuous nonlinearities are concerned, some interesting results are reported in [31]. The papers of the authors in

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https://doi.org/10.1016/j.cnsns.2017.11.029
1007-5704/

[32–37] talk about observer design or fuzzy control with circle criteria, and the studies of the authors in [40,41] present results for multi-agents consensus with coupled nonlinearities.

Different from circle criteria, Popov criteria are suitable in dealing with time-invariant, channel-decoupled sector nonlinearities with Popov plots [38,39] (determined by the Luré–Postnikov technique); in contrast, circle criteria rely directly on Nyquist loci. In this sense, both circle and Popov criteria are frequency-domain tools for evaluating stability robustness in the Luré problem. Indeed, such frequency-domain expression makes it possible to exploit the frequency-domain techniques about stabilization and controller design in Luré systems. Main difficulties when applying circle and Popov criteria include: (i). if used in their conventional fashion, the stability conditions must be employed in a case-by-case fashion, according to open-loop pole distribution (thus prior stability analysis is indispensable) and scalar/multivariable configuration; (ii). graphical plotting of frequency-domain features is unavoidable so that it is not so significant as an analytical tool; and (iii). if interpreting the stability conditions via LMIs, frequency-domain features are neglected largely, though LMIs are numerically tractable via the inner point algorithms.

This paper re-visits the Luré problem in light of Popov criteria [29,42,43] for absolute stability with what we call complex scaling stability loci, instead of Popov plots. This approach entails no direct stability analysis of the linear subsystems in Luré systems [30,44]. Several complex scaling Popov criteria are summarized, which uniformly accommodate both multivariable and scalar cases and disregard open-loop pole distribution. Moreover, the stability conditions are implementable graphically with the complex scaling stability loci, or numerically involving neither locus plotting nor LMI-like inequality solving. Thus, the approach is highly numerically tractable in stability analysis and stabilization design with additional parameter freedom. The complex scaling Popov criterion reveals also interesting frequency-domain facts about Luré systems that remain unknown up to now; for example, frequency spectrum in terms of positive realness and sector nonlinearities, and role of the H_∞ performance for stabilizing Luré systems.

Notations: $\mathcal{R}$ and $\mathcal{C}$ represent, respectively, the set of all real or that of all complex numbers. $\langle\cdot\rangle_k$ denotes the k-th leading principal minor. det($\cdot$) means the determinant of ($\cdot$). The degree of a polynomial $\theta(s)$ is meant by deg($\theta(s)$). $(\cdot)^*$ means the conjugate transpose of ($\cdot$). I_k denotes the $k \times k$ identity matrix. $\lambda(\cdot)$ denotes the set of all eigenvalues of ($\cdot$).

Outline: Section 2 lists preliminaries to the complex scaling stability analysis in the LTI setting. Section 3 interprets the standard Popov criterion according to the complex scaling stability loci, together with observations about the Luré systems and their LTI embedding. Section 4 sketch numerical examples, while conclusions are given in Section 5.

2. Complex scaling stability criterion

2.1. Feedback configuration and problem formulation

Let Σ represent the LTI model given by

$$\Sigma: \begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{1}$$

where $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times m}$, and $C \in \mathcal{R}^{l\times n}$; $x \in \mathcal{R}^n$ is the state vector, while $u \in \mathcal{R}^m$ and $y \in \mathcal{R}^l$ are the input and output vectors, respectively. The transfer function of Σ is written by $G(s) = C(sI_n - A)^{-1}B \in \mathcal{C}^{l\times m}$

Next, introduce the static output feedback $u = r - Ky$ to (1), where $K \in \mathcal{R}^{m\times l}$ is a gain matrix and r a new input. The state-space equation for the closed-loop system is

$$(\Sigma, K): \begin{cases} \dot{x} = [A - BKC]x + Br \\ y = Cx \end{cases} \tag{2}$$

It is said that the closed-loop system (Σ, K) is asymptotically stable, if all eigenvalues of $A - BKC$ have negative real parts.

To address asymptotical stability of (Σ, K), the return difference relationship is claimed as

$$\frac{\det(sI_n - A + BKC)}{\det(sI_n - A)} = \det(I_m + KG(s)) \tag{3}$$

By definition, $\det(sI_n - A + BKC)$ and $\det(sI_n - A)$ are the closed- and open-loop characteristic polynomials, respectively. Unfortunately, however, if there exists any factor cancelation between $\det(sI_n - A)$ and $\det(sI_n - A + BKC)$, only a coprime portion is left after all reducible factors are removed. In reducible cases, (3) reduces to a partial relationship between the closed- and open-loop characteristic polynomials so that asymptotic stability cannot be claimed rigorously.

To address asymptotical stability of (Σ, K) even if (3) is reducible, we re-write (3) equivalently into the complex scaling return difference relationship

$$\frac{\det(sI_n - A + BKC)}{\theta(s)} = \frac{\det(sI_n - A)}{\theta(s)} \det(I_m + KG(s)) \tag{4}$$

where $\theta(s)$ is an auxiliary Hurwitz polynomial. The closed- and open-loop characteristic polynomials are juxtaposed at the two sides of (4). To facilitate our arguments, we write

$$f(s; \theta, K) := \frac{\det(sI_n - A)}{\theta(s)} \det(I_m + KG(s)) \tag{5}$$