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Research paper

Dynamics of scroll waves with time-delay propagation in excitable media

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ABSTRACT

Information transmission delay can be widely observed in various systems. Here, we study the dynamics of scroll waves with time-delay propagation among slices in excitable media. Weak time delay induces scroll waves to meander. Through increasing the time delay, we find a series of dynamical transitions. Firstly, the straight filament of a scroll wave becomes twisted. Then, the scroll wave breaks and forms interesting patterns. With long time delay, loosed scroll waves are maintained while their period are greatly decreased. Also, cylinder waves appears. The influences of diffusively coupling strength on the time-delay-induced scroll waves are studied. It is found that the critical time delay characterizing those transitions decreases as the coupling strength is increased. A phase diagram in the diffusive coupling-time delay plane is presented.

1. Introduction

Spiral waves are often encountered in systems ranging from physical, chemical and biological systems [1–3]. Scroll waves are the three-dimensional (3D) counterparts of spiral waves, which rotate around a one-dimensional singularity, the so-called filament [4]. In the cardiac tissue, scroll waves of electrical activity play a crucial role in causing arrhythmias and tachycardias [5]. Once scroll waves break up during tachycardia, they may further evolve into ventricular and atrial fibrillation that may lead to a loss of pumping function and sudden cardiac death [6]. Therefore, studying the stability of scroll waves is one of the central problems in this field and has attached much attention in the past decades.

Propagation time delay (PTD) arising from the finite signal propagation time is frequently observed in oscillator, nonlinear circuits, nerve system and biological network [7–9]. It plays important roles in determining and characterizing the dynamical behavior in such complex media [10–12]. For example, it was found that distance-dependent time delays induced various patterns including traveling rolls, squarelike and rhombuslike patterns, spirals, and targets [13]. Experiments show that the heart consists of layers of tissues [14–16], which results in PTD when the electricity activity propagates across different layers. Evidently, the scroll wave spanning over tissues will develop twist of its filament. The dynamics of filament will greatly affected by the PTD. Consequently, various dynamical behavior may be presented. However, to our knowledge, the influences of the PTD on the scroll wave have not been studied so far, which makes this investigations meanwhile.

Experimentally, scroll waves in the Belousov–Zhabotinsky (BZ) reaction have been exposed to various gradient fields, such as illumination [17], electrical fields [18], and temperature fields [19]. The externally applied gradients can drive the

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scroll waves to drift and twist [20], control the spatial orientation and lifetime of scroll rings [21], and promote scroll wave instability [22,23]. In a sense, the influences of PTD on scroll waves can be viewed as one type of gradient fields. Zhao et al. analyzed the stability of 2D spiral waves sliced from the twisted scroll in the vertical direction [24]. They showed that the 3D problem was simplified by taking into account the diffusive coupling in the third direction as a time-delayed perturbation to the two-dimensional (2D) spiral wave, which was demonstrated perfectly by their experiment results. Thus, studying the dynamics of scroll waves through changing the degree of time delay and the strength of diffusive coupling is of great interest.

2. Simulation model

We carry out numerical simulations of the Bär model for a two-variable reaction-diffusion equations [25]. This model is frequently employed to study the dynamics of general excitable media

$$\frac{\partial u}{\partial t} = f(u, v) + D\nabla^2 u,$$
$$\frac{\partial v}{\partial t} = g(u) - v. \tag{1}$$

where $f(u, v) = \frac{u(1-u)}{\varepsilon}\left[u - \frac{v+b}{a}\right]$ and $g(u, v)$ takes the form: $g(u) = 0$, if $0 \le u < 1/3$; $g(u) = 1 - 6.75u(u-1)^3$, if $1/3 \le u \le 1$; $g(u) = 1$, if $1 < u$. The value of ϵ is chosen to be small to determines the relative time scale of the dynamics of u and v. Thus, u is the activator (fast) variable and v is the inhibitor (slow) variable. A set of parameters, $a = 0.84$, $b = 0.1$, and $\varepsilon = 0.04$, are fixed to ensure the medium is excitable. Spiral waves and scroll waves typically emerge in 2D and 3D medium, respectively.

The PTD is assumed to occur in the Z direction, i.e. among different slices. In our discretization, the PTD between neighboring layers is described by the Eq. (2)

$$\frac{\partial u_{i,j,k}}{\partial t} = f(u_{i,j,k}, v_{i,j,k}) + \frac{D_s}{\Delta r^2}(u_{i+1,j,k} + u_{i-1,j,k} + u_{i,j+1,k} + u_{i,j-1,k} - 4u_{i,j,k}) + \frac{D}{\Delta r^2}(u_{i,j,k+1}(t - \Delta\tau) + u_{i,j,k-1}(t - \Delta\tau) - 2u_{i,j,k}(t)),$$
$$\frac{\partial v_{i,j,k}}{\partial t} = g(u_{i,j,k}) - v_{i,j,k}. \tag{2}$$

where the subscript (i, j, k) denotes the grid (i, j) in the k-th layer. D_s is the diffusion coefficient in a x-y slice and its value is fixed to be 1.0. The diffusion coefficient D along the Z direction represents diffusive coupling strength between two neighboring layers and $\Delta\tau$ shows the delay time among them. We carry out the simulation through varying the values of $\Delta\tau$ and D.

The 3D system is discretized into $256 \times 256 \times 40$ lattice with spacial step $\Delta r = 0.39$. The system consists thus of 40 diffusively coupled layers of 2D reaction-diffusion systems. Simulation using finer discretization with more pixels in the vertical direction produce consistent results. The Eqs. (1)–(2) are integrated numerically using the Euler algorithm with time step $\Delta t = 0.02$ and no-flux boundary condition. For each layer of the medium, same spirals are used as the initial conditions. The diffusive coupling in the direction along the filament (Z) is applied after $t = 100$. We consider only the asymptotic behaviors after long time evolution.

3. Results and discussions

In a previous investigation, Zhao et al. studied the dynamics of scroll waves in a 3D system with gradient of excitability [24]. Theoretically, they proposed that the gradient of excitability can be simplified by taking into account the diffusive coupling in the third direction as a time-delayed perturbation to the 2D spiral wave. They found that the PTD may induce twisting of filaments, which resulted in meandering of a scroll wave. Here, we simulate a scroll wave in a 3D medium with PTD directly. Without PTD, the stable scroll wave rotates with period $T_0 = 4.4$. Trajectory of one tip in each layer draws a circle. We scale the time delay as $\Delta\tau \to \Delta\tau/T_0$.

We slightly increase the PTD ($\Delta\tau$) while keeping the coupling strength $D = 1.0$. In Fig. 1 (a) with $\Delta\tau = 0.001$, we plot the evolution of u from a fixed grid point in the 4th and 20th layers, respectively. Due to PTD, there is a phase difference at moment around $t = 200$. The corresponding filament in Fig. 1(d) shows a twisted configuration. Since the PTD is small, the phase difference disappears when the system reaches a stable state, which can be seen in Fig. 1(b). Consequently, the initially developed twisted filament gradually become a straight line, which is displayed in Fig. 1(e). The trajectories of spiral tips from the 4th and 20th show flowerlike orbits with outwards petals. One can see that they almost coincide with each other, which means the 2D spirals in different layers are in a synchronous meandering state. Therefore, the scroll wave meanders with a straight filament when PTD is slight.

When the PTD is increased gradually, a transition occurs. Obviously, the phase difference in Fig. 2(a) is kept even the system experiences long time evolution in Fig. 2(b). In this case, the filament keeps on twisting. Comparing the filament in Fig. 2(d) and (e), one can see this point. The phase difference can also been found from the trajectories in Fig. 2(c). Thus, the scroll wave meanders with a twisted filament.