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Research paper

Mean-field velocity difference model considering the average effect of multi-vehicle interaction

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ABSTRACT

In this paper, a mean-field velocity difference model(MFVD) is proposed to describe the average effect of multi-vehicle interactions on the whole road. By stability analysis, the stability condition of traffic system is obtained. Comparison with stability of full velocity-difference (FVD) model and the completeness of MFVD model are discussed. The mKdV equation is derived from MFVD model through nonlinear analysis to reveal the traffic jams in the form of the kink-antikink density wave. Then the numerical simulation is performed and the results illustrate that the average effect of multi-vehicle interactions plays an important role in effectively suppressing traffic jam. The increase strength of the mean-field velocity difference in MFVD model can rapidly reduce traffic jam and enhance the stability of traffic system.

1. Introduction

In recent years, traffic problems, such as traffic jam, fuel consumption and environmental pollution, *etc*, have become more and more serious in many cities. Many traffic phenomena like start-stop waves, fractal behavior, long-rang correlation, self-similarity and so on have attracted more and more attentions [1–3]. In order to deeply understand them, traffic models such as fluid-dynamical models, kinetic models, car-following models and cellular automaton (CA) models, etc. have been proposed [1–3]. There are several typical car-following models, i.e., optimal velocity (OV) model [4], generalized force (GF) model [5], full velocity difference (FVD) model [6], coupled map (CM) model [7,8] and so on. Among them, the OV model is one of the favorable car-following models for studying traffic flow and has successfully revealed the dynamical evolution of traffic congestion and the formation of stop-and-go traffic.

In 1999, Konishi et al. [7] proposed a CM model corresponding to the discrete version of the OV model. CM model is applied to traffic control suppressing traffic jams via introducing a delayed-feedback control method. They investigated the noise effects under open boundary bottleneck conditions on computer simulations and shown that the traffic system could run well under control. Based on it, Zhao and Gao [9] presented a simple strategy for congested traffic induced by bottlenecks in the traffic system. Han et al. [10] modified the CM model by considering both the forward-and- backward looking information. In 2011, Ge et al. [11] proposed the CM model via modification of the OV function, which depends not only on the headway distance of the current vehicle but also on the preceding ones. In 2013, Zhou et al. [12] studied the influence of the feedback control scheme on traffic jam by using the CM model. From these studies, control scheme on the

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CM model or OV model have been verified to play an important role in suppressing traffic jam [7–12]. It not only represents the traffic phenomena in a simple way, but also easily exerts on control scheme to realize control of traffic system [13–16].

Besides, Intelligent Transportation System (ITS) especially the real-time traffic information on whole road can be provided many effective feedback control strategies to suppress traffic jam. The adaptive cruise control (ACC) system is one of the recent advances in ITS [17–18,23–25,27]. ACC allows drivers to follow each other automatically, in which information of each vehicle is almost determined. It has been reported that if 20% of vehicles were equipped with ACC systems, almost all of the congestion on a German autobahn would have been eliminated [23]. Swaroop and Rajagopal [24] and Yi and Horowitz [25] proposed the macroscopic models for ACC traffic systems. Ngoduy [27] extended the multiclass gas-kinetic theory of Hoogendoorn and Bovy [26] to ACC traffic systems via taking the speed variance of ACC vehicles into account. Moreover, ITS can make the best of traffic model of the multi-vehicle interaction to predict traffic. Although the interaction among vehicles on road is different from each other and it depends on the headway in front and behind, relative velocity and other factors, the average effect of interaction of many vehicles on the whole road can be considered in the light of the mean-field idea. For multi-vehicle interactions problem, there are many related traffic models like the multiple velocity difference model(MVD), multiple "look-ahead" model(MLA) and so on [19,20], these models just consider the effect of the velocity difference of some vehicles in front, the relative velocity, headway of some vehicles with different weight or effect of looking at the vehicle that follows based on OV model. In traffic model of [20], a driver looks at the following vehicle as well as the preceding vehicle. It is found the effect of looking back at the vehicle that follows and show that this effect effectively stabilizes the traffic flow. Ge et al. [21] and Shi et al. [22] proposed the multiple velocity traffic model via focusing on the several preceding vehicles with different weight. These traffic models affect the stability of traffic flow and the formation of density wave via linear and nonlinear stability analysis. As a matter of fact, many factors or effects are taken into account during traffic modelling, which can contribute to the development of traffic control.

In this paper, we attempt to discover the average effect of all vehicle interactions on the whole road to suppress the traffic jams. Different to the FVD model, we propose a mean-field velocity difference model (MFVD) to stabilize the traffic by using the mean-field interaction term rather than the front vehicle's velocity in the FVD model. The details description of the model which considers the average effect of all vehicle interactions on traffic flow is explained in Section 2. In Section 3, we carry out the linear stability analysis and discuss the characteristics of MFVD model. In Section 4, the mKdV equation is derived from MFVD model to describe traffic jamming in the form of the kink-antikink density wave by nonlinear analysis. In Section 5, we perform the numerical simulations to test the effectiveness of suppressing the traffic jam. Finally, some conclusions are yielded in Section 6.

2. Mean-field velocity difference (MFVD) model

In 1995, Bando etc. [4] proposed a car-following model call optimal velocity model (OV model). In OV model, an optimal velocity function reflects drivers to adjust his velocity to the optimal value avoiding collision within delay time. The OV model can well describe the formation of traffic jam like stop & go traffic. The OV model is as follows:

$$\frac{d^2x_j(t)}{dt^2} = a\left(V_e(\Delta x_j(t)) - \frac{dx_j(t)}{dt}\right) \tag{1}$$

where $x_j(t)$ is the position of the jth vehicle, $\Delta x_j(t) = x_{j+1}(t) - x_j(t)$ is the headway of jth vehicle at time t, and a is sensitivity coefficient of drivers. The OV function reads as follows

$$V_e(\Delta x_j(t)) = \frac{V_{\max}}{2}(tanh(\Delta x_j(t)) - h_c) + tanh(h_c)) \tag{2}$$

where $V_{\max}$ is the maximal velocity and h_c is the safety distance. Bando et al. proved that the stability condition is

$$\frac{2V'(h_c)}{a} < 1 \ or \ f < \frac{a}{2} \tag{3}$$

where $V'(h) = f = dV(\Delta x_j)/d\Delta x_j \ |\Delta x_j = h_c$, and h_c is the mean headway. If the stability condition does not hold, traffic flow is in an unstable region, small deviation from the uniform traffic flow will induce stop-and-go traffic phenomenon. However, OV model encounters some problems of too high acceleration and unrealistic deceleration [5] and GF model does not take the effect of positive on traffic dynamics into account [6]. In 2001, Jiang et al. [6] studied these drawbacks and proposed FVD model only considering relative velocity (velocity difference). FVD model can better describe the phase transition of traffic flow and estimate the evolution of traffic congestion. Although the interaction of vehicles on road is different from each other, which is associated with headway in front and behind, relative velocity and other factors, the interaction exerting on each vehicle can be taken the place of an average force of all vehicle interactions on the whole road according to the mean-field theory. That is a mean field coupling term is introduced into the full velocity difference (FVD) model to replace the front vehicle's velocity. Thus, a mean-field velocity difference model (MFVD) is proposed as follows.

$$\frac{d^2x_j(t)}{dt^2} = a(V_e(\Delta x_j(t)) - \frac{dx_j(t)}{dt}) + ak(\frac{1}{n}\sum_{l=0}^{n-1}\frac{dx_{j+l}(t)}{dt} - \frac{dx_j(t)}{dt}). \tag{4}$$