Commun Nonlinear Sci Numer Simulat 000 (2017) 1–12

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Dynamics of a minimal consumer network with *bi*-directional influence

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ARTICLE INFO

Article history:
Available online xxx

Keywords:
Two-dimensional noninvertible map
Coexistence of attractors
Basin of attraction
Bifurcation diagram

ABSTRACT

We study the dynamics of a model of interdependent consumer behavior defined by a family of two-dimensional noninvertible maps. This family belongs to a class of coupled logistic maps with different nonlinearity parameters and coupling terms that depend on one variable only. In our companion paper we considered the case of independent consumers as well as the case of *uni*-directionally connected consumers. The present paper aims at describing the dynamics in the case of a *bi*-directional connection. In particular, we investigate the bifurcation structure of the parameter plane associated with the strength of coupling between the consumers, focusing on the mechanisms of qualitative transformations of coexisting attractors and their basins of attraction.

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1. Introduction

The economic model considered in the present paper is defined by a family of two-dimensional (2D for short) *noninvertible* maps. In modeling of real world processes in various fields of science maps of this class appear quite often. The research direction associated with *Nonlinear Economic Dynamics* is one of such fields which has been rapidly developing in the last decades. In fact, great achievements in the nonlinear dynamics theory combined with new numerical tools and methods allowed not only to revise some classical economic models adding more realistic assumptions, but also to introduce intrinsically new models taking into account such important features as ability of agents to learn and to adapt, their bounded rationality and heterogeneity, incomplete information, and so on. From the mathematical point of view such models are mainly nonlinear, depending on many parameters, and as a rule they allow for multiple equilibria as well as other, more complex, coexisting attractors, complicated basins of attraction, diverse bifurcation scenarios observed under parameter variation. Thus by means of these models many intricate phenomena can be successfully studied. An array of examples from economics and finance can be found in the following monographs [1–5].

A body of theoretical work embracing the idea of interdependence among consumers evolved in economics during the late Seventies [6–14]. Most of this work focuses on the existence of equilibria and their characterization. A dynamic analysis of the model of interdependent consumer choice was eventually provided in [15,16].

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http://dx.doi.org/10.1016/j.cnsns.2017.04.007
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The 2D noninvertible map which defines this demand model belongs to a class of coupled logistic maps which usually are considered in a symmetric (with respect to the main diagonal of the phase plane) case with various types of linear or nonlinear coupling, being related to the problem of synchronization of chaotic attractors (see, e.g. [17–23]). However, our map in general is nonsymmetric: the dynamics of its variables are governed by maps topologically conjugate to the logistic map with two different parameters of nonlinearity. Moreover, the coupling terms, which are nonlinear, depend only on the other variable. In the present paper we investigate how the asymptotic behavior of a generic trajectory of the map depends on parameters of nonlinearity of the logistic terms as well as on coupling parameters. In the economic context these parameters are referred to as learning and influence parameters respectively. They are related to the preference formation of each individual.

In [24] we have considered two decoupled maps, i.e. the coupling parameters were both set to zero, as well as the case of unilateral connection between the variables, i.e. one coupling parameter takes the value zero while the other one is positive. In the latter case the map has a triangular structure. In our present effort the dynamics of the generic case, when both coupling (influence) parameters are positive, is studied. In particular, we investigate bifurcation structure of the plane of coupling parameters for different fixed values of parameters of nonlinearity. To discuss the problem of coexistence of various attractors, regular and/or chaotic, which is typical for the considered map, we use one-dimensional (1D for short) bifurcation diagrams, as well as present examples of basins of attraction of coexisting attractors. Moreover, we recapitulate the important role of the critical lines (see [25–27]) in the investigation of dynamic properties of noninvertible maps.

The paper is organized as follows. In Section 2 we introduce the map and recap the key results concerning the dynamics of this map in case of decoupled and *uni*-directionally coupled variables. The results related to the dynamics of the map in case of *bi*-directionally coupled variables are presented in Section 3. A short conclusion found in Section 4 finalizes the paper.

2. Preliminaries

We consider two rational consumers (myopic utility maximizers) who interact in some way. It is hypothesized that each individual adjusts her preferences on the basis of her own past consumption choices as well as on the choices of the other individual. The adjustment of the preferences (Cobb-Douglas utility functions) is modeled as a discrete time non-linear dynamic system on a space constituted by the parameters of the utility function. As shown in [15,16], the dynamics of the resulting demand model is described by a family of 2D noninvertible maps $F:\mathbb{R}^2_+\to\mathbb{R}^2_+$ defined as follows:

$$F:\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto \begin{pmatrix} \frac{b_1}{p_xp_y}(\alpha_1x_1(b_1-p_xx_1)+D_{12}x_2(b_2-p_xx_2)) \\ \frac{b_2}{p_xp_y}(\alpha_2x_2(b_2-p_xx_2)+D_{21}x_1(b_1-p_xx_1)) \end{pmatrix} \tag{1}$$

where the variables x_1 and x_2 are units of a commodity x consumed by individuals 1 and 2 (at time t). The parameters b_1 and b_2 represent the exogenous incomes of individual 1 and 2, while p_x and p_y denote the prices of consumption goods x and y (by assumption, an individual spends the entire income on the consumption of the commodities x and y, therefore, the consumption dynamics can be expressed in terms of commodity x only). The remaining parameters α_1, α_2 and D_{12}, D_{21} are referred to as *learning* and *influence* parameters respectively. All parameters are real and positive. Moreover, the conditions

$$\alpha_1b_1^2+D_{12}b_2^2<4p_xp_y,\quad \alpha_2b_2^2+D_{21}b_1^2<4p_xp_y \tag{2}$$

have to be satisfied, which guarantee that the budget constraints for both individuals, that is, the conditions $0<x_1<\frac{b_1}{p_x}$ and $0<x_2<\frac{b_2}{p_x}$, are satisfied at any iteration by F. In other words, if (2) holds then $F(S)\subset S$ where

$$S=\left(0,\frac{b_1}{p_x}\right)\times\left(0,\frac{b_2}{p_x}\right) \tag{3}$$

is the feasible phase region.

In the present study we keep the parameters p_x, p_y, b_1 and b_2 fixed as follows:

$$p_x=\frac{1}{4},\quad p_y=1,\quad b_1=10,\quad b_2=20 \tag{4}$$

and investigate the dynamics of F depending on α_1, α_2, D_{12} and D_{21}. For the parameter values satisfying (4) the conditions (2) can be written as follows:

$$D_{12}<0.25(0.01-\alpha_1),\quad D_{12}<4(0.0025-\alpha_2) \tag{5}$$

and the feasible phase region is defined as

$$S=(0,40)\times(0,80). \tag{6}$$

As mentioned in the introduction two qualitatively different cases of map F have been dealt with in [24]:

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