Commun Nonlinear Sci Numer Simulat 57 (2018) 388–401

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Confidence intervals and hypothesis testing for the Permutation Entropy with an application to epilepsy

Francisco Traversaro [a,b,*], Francisco O. Redelico [c,d]

[a] *CONICET - Universidad Nacional de Lanús, Grupo de Investigación en Sistemas de Información, 29 de Septiembre 3901, Lanús, Buenos Aires B1826GLC, Argentina*

[b] *Instituto Tecnólgico de Buenos Aires (ITBA), Av. Eduardo Madero 399 - (C1181ACH) Ciudad Autónoma de Buenos Aires, Argentina*

[c] *CONICET - Hospital Italiano de Buenos Aires, Departamento de Informática en Salud, Perón 4190 - (C1199ABB) Ciudad Autónoma de Buenos Aires, Argentina*

[d] *Universidad Nacional de Quilmes, Departamento de Ciencia y Tecnología, Roque Sáenz Peña 352 - (B1876BXD) Bernal, Buenos Aires, Argentina*

ARTICLE INFO

Article history:
Received 18 May 2017
Revised 2 October 2017
Accepted 23 October 2017
Available online 24 October 2017

Keywords:
Permutation Entropy
Epilepsy
Hypothesis test

ABSTRACT

In nonlinear dynamics, and to a lesser extent in other fields, a widely used measure of complexity is the Permutation Entropy. But there is still no known method to determine the accuracy of this measure. There has been little research on the statistical properties of this quantity that characterize time series. The literature describes some resampling methods of quantities used in nonlinear dynamics - as the largest Lyapunov exponent - but these seems to fail. In this contribution, we propose a parametric bootstrap methodology using a symbolic representation of the time series to obtain the distribution of the Permutation Entropy estimator. We perform several time series simulations given by well-known stochastic processes: the $1/f^{\alpha}$ noise family, and show in each case that the proposed accuracy measure is as efficient as the one obtained by the frequentist approach of repeating the experiment. The complexity of brain electrical activity, measured by the Permutation Entropy, has been extensively used in epilepsy research for detection in dynamical changes in electroencephalogram (EEG) signal with no consideration of the variability of this complexity measure. An application of the parametric bootstrap methodology is used to compare normal and pre-ictal EEG signals.

1. Introduction

In 2002, Bandt and Pompe introduced a measure of complexity for time series [1] named *permutation entropy* (PE). It is an information entropy [2] that takes account of the time evolution of the time series, in contrast with other prominent information entropies as the Shannon entropy [3]. Its computation is fast, requires not too long time series [4], it is robust against noise [5] and distributional assumptions of the time series [6]. This measure has been widely used in non-linear dynamics [7–11], and to a lesser extent in Stochastic Processes [12–14], among others. It has also had a great impact on such different and important areas of applied science and engineering as varied as Mechanics Engineering [15,16], Epilepsy

* Corresponding author at: Instituto Tecnólgico de Buenos Aires (ITBA), Av. Eduardo Madero 399 - (C1181ACH) Ciudad Autónoma de Buenos Aires, Argentina.

E-mail address: francisco.redelico@hospitalitaliano.org.ar (F. Traversaro).

https://doi.org/10.1016/j.cnsns.2017.10.013

[17,18], Anesthesia [19], Cardiology [20,21], Finance [22], Climate Change [23]. Also a book about Permutation Complexity has been published [24].

Since its publication and up to the end of 2016, the Bandt and Pompe seminal work [1] has been cited in 789 papers, according Scopus bibliographic database, and the evolution of the cites seems to indicate that it will be increasing within time. All these facts made an investigation of PE from the statistic point of view an important issue.

There has been little research, up to our knowledge, on the statistical properties of the quantities used in nonlinear dynamics to characterize time series. This lack of research may be due the lack of distributional theory of these quantities, yielding resampling techniques as the most powerful tool to overcome this task. Perhaps one exception to this is the research on the distribution of the largest Lyapunov exponent and the correlation dimension [25]. In [26] a methodology to calculate the empirical distributions of Lyapunov exponents based on a traditional bootstrapping technique is presented, providing a formal test of chaos under the null hypothesis. However, in [27] it is shown that the previously bootstrap approach seems to fail to provide reliable bounds for the estimates of the Lyapunov's exponents, and conclude that the traditional bootstrap cannot be applied for estimating multiplicative ergodic statistics. In [28], a moving blocks bootstrap procedure is used to detect a positive Lyapunov exponent in financial time series. However, the time series generated by moving block bootstrap present artifacts which are caused by joining randomly selected blocks, so the serial dependence is preserved within, but not between, the blocks.

Regarding time series symbolic dynamics, in [29] the probabilities generated using the Bandt and Pompe methodology are calculated analytically for Gaussian Processes for motiv length equal three, but they recognize that for larger lengths this is not possible, and for that reason a computer based method is required to estimate the bias and variance in the PE estimation.

In this contribution we propose a different simulation method (i.e. parametric bootstrap) for estimating the bias, variance and confidence intervals for the Permutation Entropy estimation, along with hypothesis testing, that consists in simulating symbolic time series bootstrap samples that are thought to be produced by a probabilistic model with a fixed transition probability extracted from the original time series.

In order to show some results from our method we simulate a well known family of time series: the $1/f^{\alpha}$ noise. We compute bias, variance and confidence intervals for the Permutation Entropy of these time series according to the time series length along with other parameters. In addition, an application of the parametric bootstrap methodology for hypothesis testing is used to compare normal and pre-ictal EEG signals.

The paper reads as follows: Section 2 shows a brief review of PE in order to present the estimator to be evaluated using the bootstrap approach, Section 3 presents and explains the proposed parametric bootstrap, firstly a brief review of the bootstrap scheme is done as an introduction to our method, then in Section 3.1 the core of this bootstrap approach is presented, i.e. the probability transitions computation, and finally in Section 3.2 the algorithms to perform the parametric bootstrap of PE is explained. Section 4 addresses the numerical simulation along with its results, in Section 5 an application of the parametric bootstrap is used to compare normal and pre-ictal EEG signals and Section 6 is devoted to the conclusions of this contribution.

2. Permutation Entropy

In this Section, we briefly review the PE to make the article self-contained and accessible for a wider audience.

Let $\{X_t\}_{t\in T}$ be a realization of a data generator process in form of a real valued time series of length $T \in \mathbb{N}$. A measure of uncertainty about $\{X_t\}_{t\in T}$ is the *normalized* Shannon entropy [3] ($0 \le \mathcal{H} \le 1$), which is defined as:

$$\mathcal{H}[P] \;=\; S[P]/S_{max} \;=\; \left\{-\sum_{i=1}^{N} P_i \;\ln(P_i)\right\}/S_{max}, \tag{1}$$

where P_i is a probability to be extracted from the time series, N is the cardinality of the P_i set $\{p_i\}_1^N$, the denominator $S_{max} = S[P_e] = \ln N$ is obtained by a uniform probability distribution $P_e = \{P_i = 1/N,\ \forall i = 1, \cdots, N\}$.

Bandt and Pompe proposed a symbolization technique to estimate P_i and compute PE, $\hat{\mathcal{H}}(m, \tau)$. First, we recall that PE has two tuning parameters, i.e. m the symbol length and τ the time delay. Within this paper, we set $\tau = 1$ with no loss of generality and it will be omitted, so we will use $\mathcal{H} = \mathcal{H}(m)$ for sake of simplicity. Let $X_m(t) = (x_t, x_{t+1}, \ldots, x_{t+m-1})$ with $0 \le t \le T - m + 1$ be a non-disjoint partition containing the vectors of real values of length m of the time series $\{X_t\}_{t\in T}$. Let $S_{m\ge 3}$ the symmetric group of order $m!$ formed by all possible permutations of order m, $\pi_i = (i_1, i_2, \ldots, i_m) \in S_m$ ($i_j \ne i_k \forall j \ne k$ so every element in π_i is unique). We will call an element π_i in S_m a symbol or a motiv as well. Then $X_m(t)$ can be mapped to a symbol π_i in S_m for a given but otherwise arbitrary t. The m real values $X_m(t) = (x_t, x_{t+1}, \ldots, x_{t+m-1})$ are mapped onto their rank. The rank function is defined as:

$$R(x_{t+n}) = \sum_{k=0}^{m-1} \mathbb{1}(x_{t+k} \le x_{t+n}) \tag{2}$$

where $\mathbb{1}$ is the indicator function (i.e $\mathbb{1}(Z) = 1$ if Z is true and 0 otherwise), $x_{t+n} \in X_m(t)$ with $0 \le n \le m-1$ and $1 \le R(x_{t+n}) \le m$. So the rank $R(min(x_{t+k})) = 1$ and $R(max(x_{t+k})) = m$. The complete alphabet is all the possible permutations