12th IFAC/IEEE Workshop on Discrete Event Systems
Cachan, France. May 14-16, 2014

Infinitesimal Perturbation Analysis for Quasi-Dynamic Traffic Light Controllers ⋆

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Abstract: We consider the traffic light control problem for a single intersection modeled as a stochastic hybrid system. We study a quasi-dynamic policy based on partial state information defined by detecting whether vehicle backlogs are above or below certain controllable thresholds. Using Infinitesimal Perturbation Analysis (IPA), we derive online gradient estimators of a cost metric with respect to these threshold parameters and use these estimators to iteratively adjust the threshold values through a standard gradient-based algorithm so as to improve overall system performance under various traffic conditions. Results obtained by applying this methodology to a simulated urban setting are also included.

Keywords: stochastic flow model (SFM), perturbation analysis, stochastic hybrid system (SHS), traffic light control.

1. INTRODUCTION

The Traffic Light Control (TLC) problem consists of adjusting green and red light cycles in order to control the traffic flow through an intersection and, more generally, through a set of intersections and traffic lights. The ultimate objective is to minimize congestion (hence delays experienced by drivers) at a particular intersection, as well as an entire area consisting of multiple intersections with traffic lights. Recent technological developments have made it possible to collect and process traffic data so that they may be applied in solving the TLC problem in real time. Fundamentally, TLC is a form of scheduling for systems operating through simple switching control actions, and several different types of optimization formulations, along with their corresponding solution algorithms, have been proposed for it (the reader is referred to [Geng and Cassandras (2012)] for a review of existing algorithms). Perturbation analysis techniques were used by Head et al. (1996) and Fu and Howell (2003) for modeling a traffic light intersection as a stochastic Discrete Event System (DES), while an Infinitesimal Perturbation Analysis (IPA) approach, using a Stochastic Flow Model (SFM) to represent the queue content dynamics of roads at an intersection, was presented in [Panayiotou et al. (2005)].

Our work is also based on modeling traffic flow through an intersection controlled by switching traffic lights as an SFM, which conveniently captures the system's inherent hybrid nature: while traffic light switches exhibit event-driven dynamics, the flow of vehicles through an intersection is best represented through time-driven dynamics. In [Geng and Cassandras (2012)], IPA was applied with respect to controllable green and red cycle lengths for a single isolated intersection and in [Geng and Cassandras (2013a)] for multiple intersections. Traffic flow rates need not be restricted to take on deterministic values, but may be treated as stochastic processes (see [Cassandras et al. (2002)]), which are suited to represent the continuous and random variations in traffic conditions. Using the general IPA theory for Stochastic Hybrid Systems (SHS) in [Wardi et al. (2010)] and [Cassandras et al. (2010)], on-line gradients of performance measures are estimated with respect to several controllable parameters with only minor technical conditions imposed on the random processes that define input and output flows. These IPA estimates have been shown to be unbiased, even in the presence of blocking due to limited resource capacities and of feedback control (see [Yao and Cassandras (2011)]).

In contrast to earlier work where the adjustment of light cycles did not make use of real-time state information, Geng and Cassandras (2013b) proposed a quasi-dynamic control setting in which partial state information is used conditioned upon a given queue content threshold being reached. In this paper, we draw upon this setting, but rather than controlling the light cycle lengths as in [Geng and Cassandras (2013b)], here we focus on the threshold parameters and derive IPA performance measure estimators necessary to optimize these parameters, while assuming fixed cycle lengths. Our goal is to compare the relative effects of the threshold parameters and the light cycle length parameters on our objective function, build upon these results, and ultimately control both the light cycle lengths and the queue content thresholds simultaneously.

In Section 2, we formulate the TLC problem for a single intersection. Section 3 details the derivation of an IPA estimator for the cost function gradient with respect to a controllable parameter vector defined by these thresholds. The IPA estimator is then incorporated into a gradient-

⋆ The authors' work is supported in part by NSF under Grant CNS-1139021, by AFOSR under grant FA9550-12-1-0113, by ONR under grant N00014-09-1-1051, and by ARO under Grant W911NF-11-1-0227.

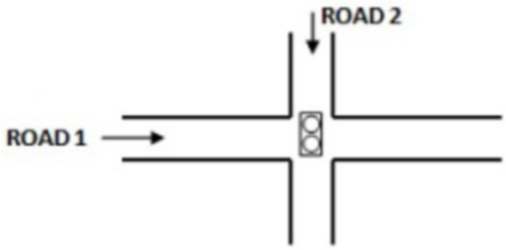

Fig. 1. A single traffic light intersection with two cross-roads

based optimization algorithm and we include simulation results in Section 4.

2. PROBLEM FORMULATION

The system we consider comprises a single intersection, as shown in Fig. 1. For simplicity, left-turn and right-turn traffic flows are not considered and yellow light cycles are implicitly accounted for within a red light cycle. We assign to each queue i a guaranteed minimum GREEN cycle length $\theta_{i,\min}$, and a maximum length $\theta_{i,\max}$ which (in contrast to [Geng and Cassandras (2013b)]) we assume to be fixed. We define a state vector $x(t) = [x_1(t), x_2(t)]$ where $x_i(t) \in \mathbb{R}^+$ is the content of queue i. For each queue i, we also define a "clock" state variable $z_i(t)$, $i = 1, 2$, which measures the time since the last switch from RED to GREEN of the traffic light for queue i, so that $z_i(t) \in [0, \theta_{i,\max}]$. Setting $z(t) = [z_1(t), z_2(t)]$, the complete system state vector is $[x(t), z(t)]$. Within the *quasi-dynamic* setting considered in this work, the controllable parameter vector of interest is given by $\mathbf{s} = [s_1, s_2]$, where $s_n \in \Re^+$ is a queue content threshold value for road $n = 1, 2$. The notation $x(\mathbf{s}, t) = [x_1(\mathbf{s}, t), x_2(\mathbf{s}, t)]$ is used to stress the dependence of the state on these threshold parameters. However, for notational simplicity, we will henceforth write $x(t)$ when no confusion arises; the same applies to $z(t)$.

Let us now partition the queue content state space into the following four regions: $X_0 = \{(x_1, x_2) : x_1(t) < s_1, x_2(t) < s_2\}$; $X_1 = \{(x_1, x_2) : x_1(t) < s_1,\ x_2(t) \geq s_2\}$; $X_2 = \{(x_1, x_2) : x_1(t) \geq s_1,\ x_2(t) < s_2\}$; $X_3 = \{(x_1, x_2) : x_1(t) \geq s_1,\ x_2(t) \geq s_2\}$. At any time t, the feasible control set for the traffic light controller is $U = \{1, 2\}$ and the control is defined as:

$$u\left(x(t), z(t)\right) \equiv \begin{cases} 1 & \text{i.e., set road 1 GREEN, road 2 RED} \\ 2 & \text{i.e., set road 2 GREEN, road 1 RED} \end{cases} \tag{1}$$

A *dynamic* controller is one that makes full use of the state information $z(t)$ and $x(t)$. Obviously, $z(t)$ is the controller's known internal state, but the queue content state is generally not observable. We assume, however, that it is *partially* observable. Specifically, we can only observe whether $x_i(t)$ is below or above some threshold $s_i, i = 1, 2$ (this is consistent with actual traffic systems where sensors (typically, inductive loop detectors) are installed at each road near the intersection). In this context, we shall define a *quasi-dynamic* controller of the form $u\left(X(t), z(t)\right)$, with $X(t) \in \{X_0, X_1, X_2, X_3\}$, as follows:

For $X(t) \in \{X_0, X_3\}$:

$$u\left(z(t)\right) = \begin{cases} 1 & \text{if } z_1(t) \in (0, \theta_{1,\max}) \text{ and } z_2(t) = 0 \\ 2 & \text{otherwise} \end{cases} \tag{2}$$

For $X(t) = X_1$:

$$u\left(z(t)\right) = \begin{cases} 1 & \text{if } z_1(t) \in (0, \theta_{1,\min}) \text{ and } z_2(t) = 0 \\ 2 & \text{otherwise} \end{cases} \tag{3}$$

For $X(t) = X_2$:

$$u\left(z(t)\right) = \begin{cases} 2 & \text{if } z_2(t) \in (0, \theta_{2,\min}) \text{ and } z_1(t) = 0 \\ 1 & \text{otherwise} \end{cases} \tag{4}$$

This is a simple form of hysteresis control to ensure that the ith traffic flow always receives a minimum GREEN light cycle $\theta_{i,\min}$. Clearly, the GREEN light cycle may be dynamically interrupted anytime after $\theta_{i,\min}$ based on the partial state feedback provided through $X(t)$. For notational simplicity, we will write $u(t)$ when no confusion arises, as we do with $x(t)$, $z(t)$.

The stochastic processes involved in this system are defined on a common probability space (Ω, F, P). The arrival flow processes are $\{\alpha_n(t)\}$, $n = 1, 2$, where $\alpha_n(t)$ is the instantaneous vehicle arrival rate at time t. The departure flow process on road n is defined as:

$$\beta_n(t) = \begin{cases} h_n(X(t), z(t), t) & \text{if } x_n(t) > 0 \text{ and } u(t) = n \\ \alpha_n(t) & \text{if } x_n(t) = 0 \text{ and } u(t) = n \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where $h_n(X(t), z(t), t)$ is the instantaneous vehicle departure rate at time t; for notational simplicity, we will write $h_n(t)$ when no confusion arises. We can now write the dynamics of the state variables $x_n(t)$ and $z_n(t)$ as follows, where we adopt the notation $\overline{n}$ to denote the index of the road perpendicular to road $n = 1, 2$, and note that the symbols t^+ (t^-, respectively) denote the time instant immediately following (preceding, respectively) time t:

$$\dot{x}_n(t) = \begin{cases} \alpha_n(t) & \text{if } z_n(t) = 0 \\ 0 & \text{if } x_n(t) = 0 \text{ and } \alpha_n(t) \leq h_n(t) \\ \alpha_n(t) - h_n(t) & \text{otherwise} \end{cases} \tag{6}$$

$$\dot{z}_n(t) = \begin{cases} 1 & \text{if } z_{\overline{n}}(t) = 0 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

$$\begin{aligned} & z_n(t^+) = 0 \text{ if } z_n(t) = \theta_{n,\max} \\ & \text{or } z_n(t) = \theta_{n,\min},\ x_n(t) < s_n,\ x_{\overline{n}}(t) \geq s_{\overline{n}} \\ & \text{or } z_n(t) > \theta_{n,\min},\ x_n(t^-) > s_n, x_n(t^+) = s_n, x_{\overline{n}}(t) \geq s_{\overline{n}} \\ & \text{or } z_n(t) > \theta_{n,\min},\ x_n(t) < s_n, x_{\overline{n}}(t^-) < s_{\overline{n}}, x_{\overline{n}}(t^+) = s_{\overline{n}} \end{aligned}$$

In this context, the traffic light intersection in Fig. 1 can be viewed as a hybrid system in which the time-driven dynamics are given by (6)-(7) and the event-driven dynamics are associated with light switches and with events that cause the value of $x_n(t)$ to change from strictly positive to zero or vice-versa. It is then possible to derive a Stochastic Hybrid Automaton (SHA) model as in [Geng and Cassandras (2013b)] containing 14 modes, which are defined by combinations of $x_n(t)$ and $z_n(t)$ values. The event set for this SHA is $\Phi_n = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$, where e_1 is the guard condition $[x_n = s_n$ from below$]$; e_2 is the guard condition $[x_n = s_n$ from above$]$; e_3 is the guard condition $[z_n = \theta_{n,\min}]$; e_4 is the guard condition $[z_n = \theta_{n,\max}]$; e_5 is the guard condition $[x_n = 0$ from above$]$; e_6 is a switch in the sign of $\alpha_n(t) - h_n(t)$ from non-positive to strictly