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Research paper

Modeling meniscus rise in capillary tubes using fluid in rigid-body motion approach

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ABSTRACT

In this study, a new term representing net flux rate of linear momentum is introduced to Lucas–Washburn equation. Following a fluid in rigid-body motion in modeling the meniscus rise in vertical capillary tubes transforms the nonlinear Lucas–Washburn equation to a linear mass-spring-damper system. The linear nature of mass-spring-damper system with constant coefficients offers a nondimensional analytical solution where meniscus dynamics are dictated by two parameters, namely the system damping ratio and its natural frequency. This connects the numerous fluid-surface interaction physical and geometrical properties to rather two nondimensional parameters, which capture the underlying physics of meniscus dynamics in three distinct cases, namely overdamped, critically damped, and underdamped systems. Based on experimental data available in the literature and the understanding meniscus dynamics, the proposed model brings a new approach of understanding the system initial conditions. Accordingly, a closed form relation is produced for the imbibition velocity, which equals half of the Bosanquet velocity divided by the damping ratio. The proposed general analytical model is ideal for overdamped and critically damped systems. While for underdamped systems, the solution shows fair agreement with experimental measurements once the effective viscosity is determined. Moreover, the presented model shows meniscus oscillations around equilibrium height occur if the damping ratio is less than one.

1. Introduction

Meniscus rise is vital in nature particularly in the transport of fluids in plants and through soil. Better understanding of meniscus dynamics is needed in several engineering applications such as inkjet printing [1], two phase loop heat pipe [2], measuring kinematic viscosity [3], fiber-base sensor [4], liquid phase in fuel cell, sponge wetting, concrete and soil wetting, candle wick wetting, wetting of fabric and food dipping. The understanding of meniscus dynamics in capillary tubes is extremely useful to understand meniscus rise in porous medium [5, 6]. Modeling the meniscus dynamics and wetting behavior in porous medium is considered challenging due to the complex nature and structure of porous media [7, 8]. Another challenge in modeling the meniscus dynamics is the effects of the invading fluid on the displaced fluid as explored by Walls et al. [9].

The challenge of understanding the physical behavior in simple cases, such as meniscus dynamics in vertical tubes, hinders extending the concept capabilities to more complex geometrical applications. In this study, the nonlinear Lucas–

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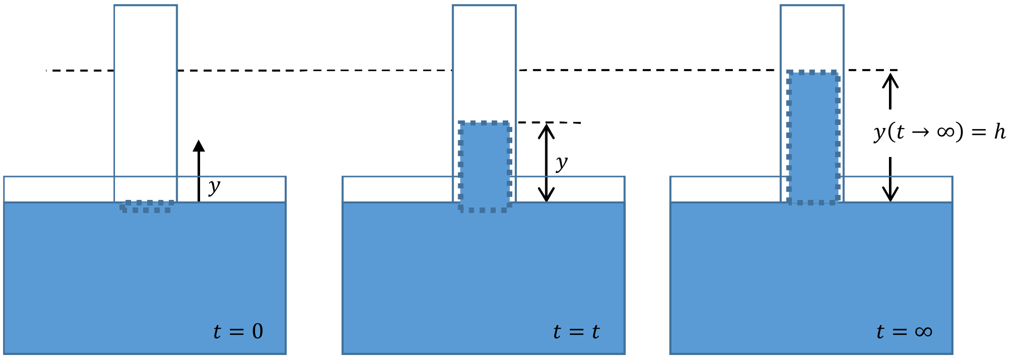

Fig. 1. Controlled volume approach used to model meniscus dynamics.

Washburn equation is transformed following a fixed-mass approach to a linear mass-spring-damper dynamic system where vertical meniscus dynamics are simply dictated by two parameters, namely the system damping ratio and its natural frequency. In addition, the proposed model brings a new way of understanding the system initial conditions in close comparison to the experimental data available in the literature.

2. Theoretical summary

The problem of meniscus rise in a capillary circular tubes dates back to the early 19th century [10–12]. The early work of Bell and Cameron [10], Lucas [11] and Washburn [12] on meniscus dynamics reported a model of the rate of meniscus penetration into a small capillary tube. Meniscus dynamics have been modeled and experimentally tested by different research groups. Quéré [13] reported experimental data with simple model to describe meniscus rise and found that oscillations around equilibrium height occur if the liquid viscosity is low enough. In the presented model, it has been shown that oscillation occurs around equilibrium when damping ratio is less one.

Using the general integral form of the linear momentum equation with a uniform inlet velocity and a deforming control volume shown in Fig. 1, the force balance on the control volume is presented as follows [14]:

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho \vec{V} d\forall + \dot{m}\vec{V}_{out} - \dot{m}\vec{V}_{in} \tag{1}$$

By simplifying the right hand side ($\dot{m} = \rho\pi R^2 \frac{dy}{dt}$, $V_{in} = \frac{dy}{dt}$, $V_{out} = 0$ and $d\forall = \pi R^2 dy$), Eq. (1) reduces to the following equation:

$$\sum F = \rho\pi R^2\left[\frac{d}{dt}\left(y\frac{dy}{dt}\right)\right] - \left(\rho\pi R^2\frac{dy}{dt}\right)\frac{dy}{dt} \tag{2}$$

Substituting external forces such as surface tension, gravity and friction forces as shown in Fig. 1, Eq. (2) can be presented as follows:

$$2\pi R\sigma\cos\theta - \pi R^2 y\rho g - 8\pi\mu y\frac{dy}{dt} = \rho\pi R^2\left[\frac{d}{dt}\left(y\frac{dy}{dt}\right)\right] - \left(\rho\pi R^2\frac{dy}{dt}\right)\frac{dy}{dt} \tag{3}$$

In Eq. (3) the left side represents the external forces on the expanding control volume shown in Fig. 1, while the right side represents the inertia forces. The external forces are surface tension ($2\pi R\sigma\cos\theta$), gravity ($\pi R^2 y\rho g$) and viscosity ($8\pi\mu y\frac{dy}{dt}$). The inertia forces represent the time rate of change in linear momentum of the control volume and the net flux rate of linear momentum to the control volume [14]. For the selected control volume shown in Fig. 1, the net flux is represented by the inlet linear momentum since zero mass flow is leaving the selected control volume. Eq. (3) can be further simplified mathematically to the following equation [15–17]:

$$2\pi R\sigma\cos\theta - \pi R^2 y\rho g - 8\pi\mu y\frac{dy}{dt} = \rho\pi R^2\left[y\frac{d^2y}{dt^2}\right] \tag{4}$$

Under specific conditions, different analytical and numerical solutions for equation Eq. (4) exist in the literature [7, 12, 13, 16, 18, 19] and are listed in Table 1. Different researchers have used different numerical techniques [8, 17, 20, 21] to solve the vertical meniscus case. No theory exists to quantitatively predict the initial stage of imbibition in the spontaneous infiltration of a wetting liquid into capillary tubes [8]. In order to solve the nonlinear equation [17], a relaxation rate constant has been introduced. The current classical equations used to describe meniscus dynamics suffer internal inconsistencies [17]. The main source of meniscus dynamic dissipation behavior is found to be dominated at low viscosity by the entrance or exit effects [20].