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Research paper

Chimera regimes in a ring of oscillators with local nonlinear interaction

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ABSTRACT

One of important problems concerning chimera states is the conditions of their existence and stability. Until now, it was assumed that chimeras could arise only in ensembles with nonlocal character of interactions. However, this assumption is not exactly right. In some special cases chimeras can be realized for local type of coupling [1–3]. We propose a simple model of ensemble with local coupling when chimeras are realized. This model is a ring of linear oscillators with the local nonlinear unidirectional interaction. Chimera structures in the ring are found using computer simulations for wide area of values of parameters. Diagram of the regimes on plane of control parameters is plotted and scenario of chimera destruction are studied when the parameters are changed.

1. Introduction

Complex spatial structures, so-called chimeras, cause a great interest among researchers in the recent time [4–6]. Chimeras emerge in oscillatory ensembles with different dynamics of units, both regular [1,7–12] and chaotic [13–16]. They consist of alternating clusters with coherent and incoherent behavior of neighboring oscillators. One of the most important condition of chimera emergence is assumed to be the nonlocal character of unit interaction, i.e. each oscillator is linked directly with a group of neighbors. The models of ensembles with nonlocal coupling were researched in a number of studies (see, for example, the review [6]). In [17–19] the results, confirming the existence of chimera in an ensemble with global interaction were obtained. The question about possibility of chimera existence in ensembles with local coupling is yet open. Generally, chimeras disappear with interaction radius decrease. However, it is possible that chimera-like structures can exist for some cases of ensembles with local interaction. So, chimera examples were obtained in the ensemble with local inertial coupling in [2]. The coupling is introduced with help of special variable described by a linear differential equation. Besides, the metastable chimeras were observed in ensemble of harmonic oscillators in a case of local interaction [1] and in ensemble of oscillators close to a homoclinic bifurcation [3].

The special type of chimera exists in the system with delayed feedback [20,21]. This is so-called virtual chimera. It is a special regime of time intermittency, when several intervals with regular and irregular behavior alternate in a period of delay. Moreover, these intervals are practically repeated for every delay period. So, the delay period can be considered in such system as a virtual space [20,21]. Systems with delayed feedback constitute a special class of distributed dynamical systems. Instantaneous state of such system in a time moment t is defined by some function of time at the interval

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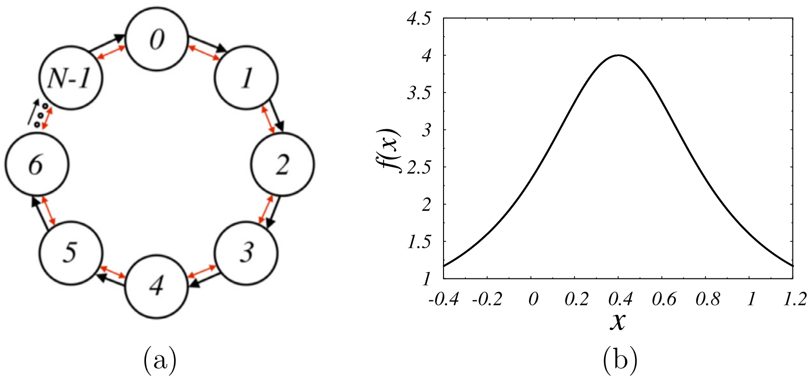

Fig. 1. (a) Schematic image of the system under study. Double arrows and one-side arrows represent diffusive coupling and unidirectional coupling, respectively; (b) graph of the nonlinear function describing the unidirectional coupling at $A = 4.7, \beta = 4, \phi = 0.4$.

$[t,\ t + T_d]$, where T_d is a delay period. Models with delayed feedback are widely used in mechanics [22], control problems [23–26], ecology [27–29], neurodynamics [30–32] and many other areas.

The behavior of oscillator with delayed feedback has similarity with one-dimensional spatially extended system. It has been shown that regime features such as the fractal dimension are extensive quantities, proportional to the time of delay, which appears to play a role very similar to the numbers of elements in a spatial system [33]. One of the main conditions of similarity between oscillator with delayed feedback and spatially distributed system is an asymmetric character of coupling between units [34]. The limit case of asymmetry is unidirectional coupling. So, it can be assumed that spatially distributed system, such as a ring of oscillators with a local unidirectional coupling under special conditions should demonstrate behavior, which is similar to the behavior of an oscillator with delayed feedback [35,36]. The existence of a virtual chimera in the systems with delayed feedback and the analogy mentioned above suggests that chimeras can emerge in a ring of oscillators with local coupling at least in a case of unidirectional character of coupling. If this hypothesis is true then the specific class of chimera structures can be obtained in the ensembles with local interaction of units. The results of numerical simulations, which prove this assumption, are presented in this article. The ensemble of linear oscillators in form of a ring with local nonlinear coupling are studied. Such system demonstrates chimeras, indeed. They are similar to the virtual chimeras in [20,21]. The evolution of chimeras is studied for variation of the system parameters. The influence of the characteristics of local coupling on the existence of the chimeric structures is considered.

2. Model of the ring

The model, studied in this work, is the ensemble consisting of identical linear dissipative oscillators. Interaction between oscillators is local and nonlinear. Boundary conditions are periodic. The schematic image of the model is shown in the Fig. 1a. The system equations are following (1):

$$\begin{cases} \dot{x}_j = -\alpha x_j - \omega_0^2 y_j + \sigma f(x_{j-1}) + \gamma (x_{j-1} + x_{j+1} - 2x_j), \\ \dot{y}_j = x_j, \end{cases}$$
$$x_{j+N} = x_j, \quad y_{j+N} = y_j. \tag{1}$$

where $j = 1, \ldots, N$ is an oscillator number (a discrete spatial coordinate), N is a number of units in the ring, α and ω_0 are parameters of oscillators (a dissipation coefficient and natural frequency, respectively). The two type of the local interactions are taken into account in the model(1). First type is a nonlinear unidirectional coupling with a force σ. Second type is a dissipative coupling with a coefficient γ. Unidirectional coupling provides the asymmetry of unit interactions and pumping of the energy into the dissipative oscillators, while the dissipative coupling leads to additional energy loss. In order to support the stationary oscillations in the ring, the unidirectional coupling force σ must be in direct ratio to dissipation α. So we can putting that $\sigma = k\alpha$, where k is a coefficient of unidirectional coupling, controlling its intensity. The nonlinearity is described by the following function (2):

$$f(x) = \frac{\beta}{1 + A\sin^2(x + \Phi)}. \tag{2}$$

This expression is known as Airy formula [37]. It describes intensity of light passing through the Fabry–Pérot interferometer. The similar function was used for setting of nonlinear delayed feedback in [21]. The parameters of nonlinearity are fixed as constant $A = 4.7, \beta = 4, \Phi = 0.4$. The graph of function $f(x)$ is shown in Fig. 1b.

The number of oscillators in the ensemble was chosen as $N = 300$. The frequency coefficient was fixed as $\omega_0 = 1$. The equations in (1) were integrated numerically using the Heun method [38]. The integration step was equal to $h = 0.0004$.