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Research paper

Line graphs for fractals

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ABSTRACT

We found the Lindenmayer systems for line graphs built on selected fractals. We show that the fractal dimension of such obtained graphs in all analysed cases is the same as for their original graphs. Both for the original graphs and for their line graphs we identified classes of nodes which reflect symmetry of the graph.

1. Introduction

Fractals are very interesting and ubiquitous objects. They are very common in nature, from blood vessels [1], through plants [2], coastlines [3] and lightnings [4] to the structure of the Universe [5]. But, as it turns out, fractals are also observed in a street network [6] or literary works [7].

One of methods which allow for a fractal construction is the Lindenmayer system [8]. It defines rules of transformation of a fractal structure from a given generation to the subsequent one. Some rules are presented in [8–10], from among which we chose three examples of three different fractal categories: Cyclical, linear and higher orders. The last category means that when we present fractal as a graph there are nodes with degree k higher than 2. For each obtained graph we then construct its line graph [11]. Nodes in a line graph constructed for a given graph replace its edges, and two nodes in a new graph are connected if they have a common node in the original graph. The question we ask here is if the obtained line graphs show fractal properties, as do graphs on which they are constructed. In this case we should be able to find the Lindenmayer system which allows for the line graph construction. We are also interested in checking the fractal dimension of the obtained line graphs.

In our earlier paper [12] some symmetric fractals were analysed in terms of the number of classes of nodes. The concept of classes was proposed by us in [13] to express symmetry of the network, as class is formed by a set of nodes with the same structure of connection with other nodes in the network. Now, the method is applied for graphs and their line graphs constructed on fractals to compare their symmetry.

The paper is organised as follows: In the next section we present line graphs obtained for some fractals and the Lindenmayer systems found for them. The following section is devoted to the class identification method applied for analysed graph and line graphs. The last section concludes the obtained results.

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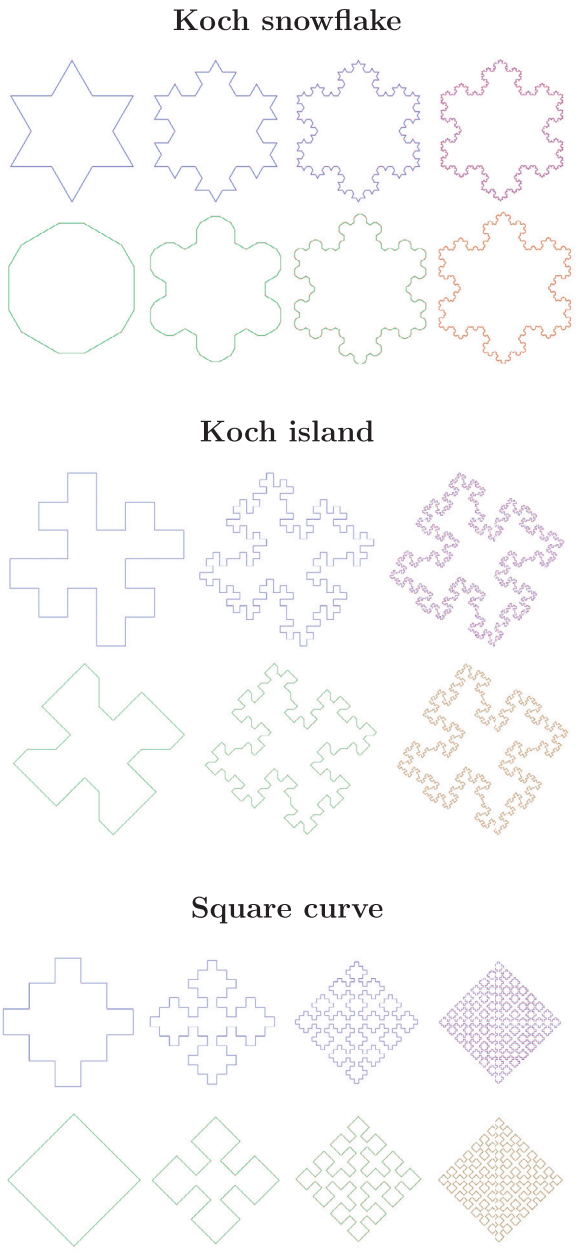

Fig. 1. Graphs (top) and it's corresponding line graphs (bottom) for cyclic fractals.

2. Line graphs and lindenmayer systems

As it was mentioned in the Introduction we focused on three types of fractals: Cyclic, linear and higher orders. From each category, three examples are chosen: Koch snowflake [9], Koch island [8], and Square curve [10] which are cyclical; Koch curve [10], Minkowski sausage [9], and Sierpinski arrowhead curve [9] which are linear, and for higher orders - Sierpinski triangle [10], Koch anti-snowflake [9], and Modified Koch curve [10].

Constructing line graphs, positions of nodes in line graphs are taken at the geometrical center of edges in original graphs. Original graphs and their line graphs analysed in the paper are presented in Figs. 1–3. While in original graphs lengths of all edges are equal, in their line graphs their are not. For all analysed here fractals, an angle between subsequent edges in the line graph is equal to half of the angle in the original graph. For such constructed line graphs we found their Lindenmayer systems. The rules obtained by us for line graphs, together with the rules for original fractals [8–10] are presented in Tables 1–3. Besides Lindenmayer rules for all analysed graphs we also calculated fractal dimension, which is defined