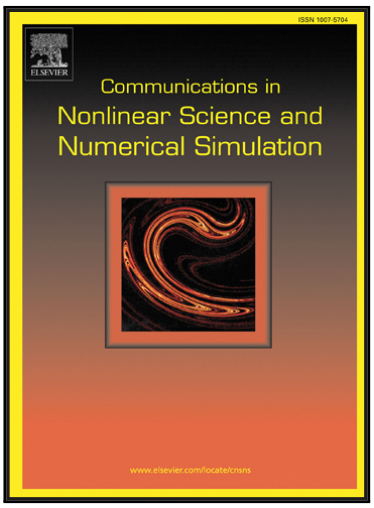

A new dispersion-relation preserving method for integrating the classical Boussinesq equation

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A new dispersion-relation preserving method for integrating the classical Boussinesq equation

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Highlights

・We derive a new system of nonlinear integral equations.

・It is equivalent to the classical Boussinesq equation.

・We apply the fixed point approach to the nonlinear system.

・It gives an iterative solution procedure (dispersion-relation *preserving* method).

・The procedure works well for integrating the Boussinesq equation.

Abstract

In this paper, a dispersion-relation *preserving* method is proposed for nonlinear dispersive waves, starting from the oldest weakly nonlinear dispersive wave mathematical model in shallow water waves, i.e., the classical Boussinesq equation. It is a semi-analytic procedure, however, which preserves, as a distinctive feature, the dispersion-relation imbedded in the model equation without adding (unwelcome) numerical effects, i.e., the proposed method has the *same* dispersion-relation as the original classical Boussinesq equation. This remarkable (dispersion-relation) preserving property is proved mathematically for small wave motion in present study. The property is also numerically examined by observing both the local wave number and the local frequency of a slowly varying water-wave group. The dispersion-relation preserving method proposed here is powerful as well for observing nonlinear wave phenomena such as solitary waves and their collision. In fact, the main features of nonlinear wave characteristics are clearly seen through not only a single propagating solitary wave but counter-propagating (head-on) solitary wave collisions. They are compared with known (exact) nonlinear solutions, the results of which represent a major improvement over existing solution formulations in the literature.

Keywords: Dispersion-relation *preserving* method; The classical Boussinesq equation; Banach fixed point theorem