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Research paper

# Geometry and dynamics of fast magnetosonic wavefronts near magnetic null points

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ABSTRACT

The behavior of two-dimensional fast magnetosonic waves in the vicinity of isolated points where the magnetic field vanishes is studied analytically. The geometry of rays and wavefronts is described, and the curvature of both is found using conformal mapping techniques. These results are applied to the formation of shock waves, obtaining that shock formation is guaranteed at a finite time for any initial condition of the perturbation when the wavefront is concave and the rays tend to focus, whereas otherwise shocks occur only for a certain range of initial conditions.



## 1. Introduction

Magnetohydrodynamic waves play a major role in many astrophysical phenomena, notably in the processes of magnetic reconnection and generation of solar flares as occur in the solar corona. Among the vast observational literature we will only mention [1]. Since magnetic nulls play an important role in the classical theory of fast magnetic reconnection, a theoretical study of MHD waves in their vicinity seemed worth pursuing. Some of the former papers consider waves depending only on the distance to a simple null point [2,3]; more realistic geometries followed. In [4] it is shown how fast magnetosonic waves in a nonhomogeneous coronal plasma refract in regions of low Alfvén velocity. In a series of papers [5–7] McLaughlin, Hood and their collaborators studied the shape of MHD waves as they approach the null point. Generation of shocks, magnetic reconnection and formation of solar flares near null points are analyzed e.g. in [8,9]. In several (but not all) of these papers, it is assumed that the plasma has a low beta, which means that the kinetic pressure gradient vanishes from the momentum equation. Independently of the physical justification of this hypothesis as concerns coronal plasma, it is theoretically logical: without a variable pressure the fast magnetosonic wave travels at the Alfvén speed $c_A = B/\sqrt{\rho}$, the Alfvén wave at $|\mathbf{B} \cdot \mathbf{n}|/\sqrt{\rho}$, and the slow wave does not move at all. Thus magnetic null points, where the Alfvén velocity vanishes, act as a breaker against any MHD wave, which cannot surpass it. By contrast, a positive sound speed would allow the waves to cross

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the null, albeit more slowly. The naive intuition that zero-$\beta$ MHD waves impacting a null point would curl around it turns out to be roughly accurate.

The mathematical treatment of the problem usually starts from the ideal (non diffusive) MHD system and study numerically the evolution of MHD waves with a variety of initial and boundary conditions. While this procedure is probably the only possible one as soon as the background magnetic field presents a minimum of complexity, it is always desirable to have a fully analytic study as long as the simplicity of the magnetic geometry allows it. Accordingly we have considered the geometry of rays and wavefronts of the fast magnetosonic wave near a X-point in dimension two; to start from a static equilibrium we need the field to be current free, i.e. irrotational. In a neighborhood of the null, rays behave as logarithmic spirals; that is, their trajectory is given by

$$t \to \exp(-At)(\cos(Bt), \sin(Bt)),$$

for some $A > 0$. The shape of wavefronts is more complex as it depends on the initial condition, but a neat trick with conformal mappings enable us the see these wavefronts as the image of a family of curves transported by a family of straight lines. This fact will enable to find the curvature of both rays and wavefronts, which will be crucial in the final section, where we consider the formation of magnetosonic shocks. The key tool is an equation satisfied by the jump of the time differential of the solutions of a quasilinear hyperbolic system propagating into an equilibrium state. This has been used for some time for the purely hydrodynamic case [10,11], as well as for the magnetohydrodynamic one [12], and in full generality [13,14]. It turns out that this quantity satisfies a transport equation along the rays which has the form of a Riccati differential equation, whose solutions may tend to infinity in a finite time; this is taken as the signal of the formation of a shock wave. This is related to the blow-up of a perturbation wave of low amplitude and high frequency, as presented in [15,16] in the context of weakly nonlinear geometrical optics. In the last case the Riccati equation is substituted by a partial differential one of Burgers' type, but the time of blow-up is similar in both approaches, which also fail after formation of caustics. We will apply these techniques to see if fast waves approaching the magnetic null form shocks; much depends on the form of the wavefront, with concave ones focusing rays and creating blow-ups.

## 2. Geometry of rays and wavefronts

The fast magnetosonic wavefronts in a medium with zero velocity are the surfaces $\tau = \text{const.}$, where $\tau$ satisfies the following particular case of the eikonal equation:

$$\frac{\partial \tau}{\partial t} = \pm c_A |\nabla \tau|, \tag{1}$$

where $c_A = B/\sqrt{\rho}$ is the Alfvén velocity, $\mathbf{B}$ the magnetic field and $\rho$ the density (see e.g.[17]). Solution of (1) depends on the initial condition $\tau(0)$, and is not guaranteed to exist for all time. Let $\mathbf{n}$ be the normal vector

$$\mathbf{n} = \frac{\nabla \tau}{|\nabla \tau|}. \tag{2}$$

If we want the waves to propagate in the direction of $\mathbf{n}$ we must take the minus sign in (1). The equations satisfied by the rays $t \to \mathbf{x}(t)$ may be set in a variety of modes (see e.g. [14,16]), but since $c_A$ does not depend on $\mathbf{n}$, they may be simplified to

$$\frac{d\mathbf{x}}{dt} = c_A \mathbf{n}$$
$$\frac{d\mathbf{n}}{dt} = \mathbf{n} \cdot (\mathbf{n} \cdot \nabla c_A) - \nabla c_A. \tag{3}$$

In this case rays are orthogonal to wavefronts. The basic state is a static plane equilibrium with constant pressure (low beta approximation), which means that the magnetic field must be current free, i.e. irrotational. In this case the field is the gradient of a harmonic function. We assume that this function has a simple zero at $\mathbf{x} = \mathbf{0}$. Higher order zeroes are unlikely to occur, since they are intrinsically unstable; if $x_0$ is an isolated zero of higher order of a function $f$, $f(x_0) = 0$, a small perturbation of $f$ may split the zero in several simple ones or it may vanish altogether: think of $f(x) = x^2$, $x_0 = 0$. Thus, by making a linear change of variables we may represent the field as

$$\mathbf{B}(x, y) = (y, x) + O(x^2 + y^2), \tag{4}$$

We assume that the density is smooth and does not vanish at $\mathbf{0}$,

$$\rho = \rho_0 + O(\sqrt{x^2 + y^2}), \tag{5}$$

Let $a = 1/\sqrt{\rho_0}$. In polar coordinates, $c_A$ may be written as

$$c_A = ar + O(r^2). \tag{6}$$