Commun Nonlinear Sci Numer Simulat 43 (2017) 14–24

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Effects of asymmetrical damping on a 2 DOF quarter-car model under harmonic excitation

M. Silveira [a,*], P. Wahi [b], J.C.M. Fernandes [a]

[a] *Department of Mechanical Engineering, São Paulo State University - UNESP, Bauru, SP, 17033-360, Brazil*
[b] *Department of Mechanical Engineering, Indian Institute of Technology - Kanpur, Kanpur-208016, India*

ARTICLE INFO

Article history:
Received 20 January 2015
Revised 11 February 2016
Accepted 17 June 2016
Available online 18 June 2016

Keywords:
Asymmetrical damper
Harmonic balance
Passenger vehicles
Comfort performance

ABSTRACT

The objective of this work is to study the dynamical behavior of vehicle suspension systems employing asymmetrical viscous damping, with a focus on improving passenger comfort. Previous studies have shown that the use of asymmetrical dampers in these types of systems can be advantageous with regard to comfort of the passengers. The modeling and the behavior of a quarter-car model with asymmetrical viscous damping under harmonic excitation is presented. The response is obtained with an analytical approximation via the method of Harmonic Balance. The choice of the asymmetry ratio diminishes the effects that the uneven road causes on the displacement and acceleration of the sprung mass. Although current systems usually adopt larger damping during the expansion phase, it is shown in this work that, for lower frequencies, smaller damping in this phase results in better comfort.

1. Introduction

Suspension systems with nonlinear elements have been studied extensively, including nonlinear springs and nonlinear dampers [1–4]. Shekhar *et al.* showed that the dampers have more influence on the dynamics than the springs [5], and the difference between linear and nonlinear elements is more pronounced for more severe shock loads [6]. These nonlinear elements have been included in active and semi-active suspension systems, in which damping properties vary according to a control algorithm applied, adding or removing energy of the system by means of valves with variable orifice area or magneto-rheological fluids [7–10]. Electromechanical suspension systems have also been studied with great interest as a means of energy recovering system [11,12].

Asymmmetrical dampers are usually designed to provide significantly greater damping force during the extension period compared to the force during the compression period [13]. These different damping coefficients in extension and compression define an asymmetry ratio. The use of an asymmetrical damping induces a shift in the steady state response of the system. The magnitude of this shift depends on the asymmetry ratio and on the excitation frequency [14].

In addition to vertical motion, usually studied with quarter-car models, the pitch displacement and acceleration were shown to be smoother for a half-car model which utilizes asymmetrical dampers [15]. Also, the vertical and angular accelerations reach minimum values at different points in the presence of asymmetrical dampers. Generally, the extension force is

* Corresponding author.
E-mail address: m.silveira@feb.unesp.br (M. Silveira).

http://dx.doi.org/10.1016/j.cnsns.2016.06.029

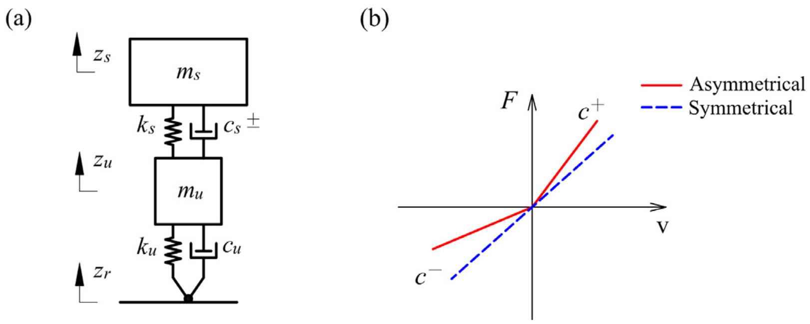

Fig. 1. Two-degree-of-freedom quarter-car model of a suspension system (a) and the generic asymmetrical characteristic of a shock absorber (b).

defined between three to four times the compressive force [16]. However, the variation of this ratio can provide very different behaviours, and has been used in control strategies [17]. The use of more stages of asymmetry has also been studied [18].

It is commonly understood and accepted that the human response to dynamic excitation depends on many mechanical, physical, physiological and psychological parameters [19]. Besides the suspension system *per se*, the seat of the vehicle is also focus of study to improve comfort in vehicles [20]. The level of comfort for the passengers depends on factors as the frequency of vibration, direction, location and time of exposure to which the passenger is subjected [20]. Apart from causing discomfort, severe vibrations may cause occupational disorders [21].

In order to further explore the use of asymmetrical damping to improve the level of comfort at different frequencies of excitation, in this paper a classical quarter-car model with 2 degrees of freedom is used with harmonic base excitation. The model is presented in Section 2; the response of the system is obtained using an analytical approximation via the method of Harmonic Balance, shown in Section 3; the convergence of the approximate solutions is illustrated in Section 4; the influence of the asymmetry ratio on the response is studied in Section 5; and conclusions are drawn in Section 6.

2. A 2-DOF quarter-car model with asymmetrical damper

There are many models suited for the dynamical analysis of vehicles, varying from simpler models to more sophisticated ones. Lower order models with lumped parameters are very common for analysis of the vertical dynamics. Quarter-car models describe exclusively vertical motion of the chassis.

Fig. 1 shows a two-degree-of-freedom shock absorption system. This system comprehends two subsystems connected in series. Each subsystem comprehends a spring and a viscous damper assembled on a structure with mass.

The base of the system represents the road on which the vehicle travels, whose irregularities cause the forces that excite the system. The lower subsystem, comprehended by the mass m_u, the spring with elastic constant k_u and the viscous damper with viscous damping coefficient c_u, represents the un-sprung mass of the vehicle, e.g. wheels, tyres, brake discs, uprights. These elements have elastic and damper characteristics that, although not being the proper suspension system, contribute to the overall dynamical behaviour of the vehicle. The upper subsystem, comprehended by m_s, k_s, and c_s, represents the proper suspension system and the sprung mass. The nonlinear absorber is adopted at this subsystem. It is clear that the behaviour of the whole system depends on the contribution of each subsystem, as on the interaction between them.

According to the classical theory of vibrations [22,23], the equation of motion for the system depicted in Fig. 1, regarding vertical displacement relative to the trivial equilibrium positions, can be written as:

$$\begin{aligned} m_u\ddot{z}_u + f_{cu} + f_{ku} - f_{cs} - f_{ks} &= 0 \\ m_s\ddot{z}_s + f_{cs} + f_{ks} &= 0 \end{aligned} \tag{1}$$

The springs are considered linear throughout this study. The spring forces f_{ku} and f_{ks} are proportional to the relative displacement, and are given by:

$$f_{ku} = k_u(z_u - z_r) \qquad\qquad f_{ks} = k_s(z_s - z_r) \tag{2}$$

The linear viscous damping force f_{cu} is proportional to the relative velocity, and is given by:

$$f_{cu} = c_u(\dot{z}_u - \dot{z}_r) \tag{3}$$

The nonlinear damping force depends on the relative velocity and its direction. In order to perform the switching of the absorber signal between the viscous damping coefficient at expansion (c_s^+) and at compression (c_s^-), representing its nonlinear characteristic, a Heaviside type function is used as follows:

$$c_s^{\pm}(\dot{z}_s - \dot{z}_u) = \begin{cases} c_s^+ & \text{if} \quad \dot{z}_s - \dot{z}_u \geq 0 \\ c_s^- & \text{if} \quad \dot{z}_s - \dot{z}_u < 0 \end{cases} \tag{4}$$