



Short communication

Lyapunov functions for a class of nonlinear systems using Caputo derivative



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ABSTRACT

This paper presents an extension of recent results that allow proving the stability of Caputo nonlinear and time-varying systems, by means of the fractional order Lyapunov direct method, using quadratic Lyapunov functions. This article introduces a new way of building polynomial Lyapunov functions of any positive integer order as a way of determining the stability of a greater variety of systems when the order of the derivative is $0 < \alpha < 1$. Some examples are given to validate these results.

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1. Introduction

Fractional Calculus, the study of integrals and derivatives of complex order, is a topic of growing interest. This is mainly due to its ability to model certain physical systems in a more adequate and precise fashion than its conventional integer order alternative [1–4].

Although some techniques to solve specific fractional order systems have been developed [1,5], the complexity of the problems has led research to the study of properties that provide qualitative but valuable information about the systems. One of these interesting properties, is the Lyapunov stability of fractional systems, which we study in this article.

The article [6] presents an extension of the Lyapunov direct method to the fractional order case. A problem that usually appears when using these techniques is the construction of an appropriate Lyapunov function to test the system's stability.

The paper [7] introduces a new way to build Lyapunov functions for nonlinear and time-varying systems when the order of the Caputo fractional derivatives is $0 < \alpha < 1$. Those Lyapunov functions are quadratic (positive definite) and are accompanied by a new Lemma that allows proving that their fractional derivatives are negative semidefinite or definite so it can be used along with the fractional extension of the Lyapunov direct method to determine stability or asymptotical stability. A similar result can be found in Corollary 1 of [8] which is a consequence of an expression, introduced in that paper, for the Caputo derivative of the product of two functions.

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These results have made possible advances in the study of dissipativity and contractivity for fractional order systems [9], Mittag-Leffler stability of fractional Lorenz-family systems [10], and Lyapunov uniform stability for fractional order systems [11]. Other relevant results presented in [12,13] allow us to easily test the stability of a Caputo fractional system of order $0 < \alpha < 1$.

Motivated by the previous articles, we extend the results of [7,8] by presenting a new Lemma for the Caputo fractional derivative, which enables us to build polynomial Lyapunov functions of greater order. We also give a similar but more general result than the Proposition 5 of [13]. Then, using these new properties, we present a partial generalization of Corollary 1 of [7] and Theorem 3.1 of [12].

The structure of this work is organized as follows: Section 2 provides a brief conceptual framework of fractional systems and their stability in the sense of Lyapunov. Section 3 introduces several new properties for the Caputo fractional derivative. Section 4 includes examples where some of the new results are used to determine the stability of certain fractional order systems. Finally, Section 5 contains this paper's conclusions.

2. Preliminary concepts

In this section, some basic fractional calculus concepts are presented, as well as the fractional order extension of the Lyapunov direct method.

In summary, fractional calculus extends the integral and differential operators to non integer order. Although various definitions of the fractional derivative are available, in this paper we will focus on the Caputo definition (1) because it's widely popular in engineering applications.

Definition 1 [7]. The Caputo fractional derivative of order $\alpha \in \mathbb{R}^+$ is defined as follows:

$${}_t^C D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad t > t_0, \quad (1)$$

where $n = \min \{k \in \mathbb{N} / k > \alpha\}$.

Theorem 1 [6]. Let $f(t, x)$ be a real-valued continuous function, defined in the domain G , satisfying in G the Lipschitz condition with respect to x , i.e.

$$\|f(t, x_1) - f(t, x_2)\| \leq l \|x_1 - x_2\|,$$

where l is a positive constant, such that

$$\|f(t, x)\| \leq M \leq \infty \quad \forall (t, x) \in G.$$

Let also

$$K \geq \frac{Mh^{\sigma_n - \sigma_1 + 1}}{\Gamma(1 + \sigma_n)}.$$

Then there exists in a region $R(h, K)$ a unique and continuous solution $y(t)$ of the following initial-value problem,

$${}_0 D_t^{\sigma_n} x(t) = f(t, x)$$

$$[{}_0 D_t^{\sigma_{k-1}} x(t)]_{t=0} = b_k \quad k = 1, 2, \dots, n,$$

where,

$$\begin{aligned} {}_a D_t^{\sigma_k} &\equiv {}_a D_t^{\alpha_k} {}_a D_t^{\alpha_{k-1}} \dots {}_a D_t^{\alpha_1}; \\ {}_a D_t^{\sigma_{k-1}} &\equiv {}_a D_t^{\alpha_{k-1}} {}_a D_t^{\alpha_{k-2}} \dots {}_a D_t^{\alpha_1}; \\ \sigma_k &= \sum_{j=1}^k \alpha_j, \quad (k = 1, 2, \dots, n); \\ 0 < \alpha_j &\leq 1, \quad (j = 1, 2, \dots, n). \end{aligned}$$

In this article we will study Caputo fractional non-autonomous systems of the form

$${}_t^C D_t^\alpha x(t) = f(t, x(t)), \quad (2)$$

where $f(t, x(t))$ is defined in Theorem 1.

An equilibrium point of the system (2) is defined as follows:

Definition 2 [4]. The constant x_0 is an equilibrium point of the fractional system (2), if and only if $f(t, x_0) = 0$.

Remark 1. For all the systems studied hereinafter, it is considered that the equilibrium points are the origin of $\mathbb{R}^n$, i.e. $x_0 = 0$. Doing so does not represent a loss of generality as any equilibrium point can be shifted to the origin through a change of variables.

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