



# The effect of process delay on dynamical behaviors in a self-feedback nonlinear oscillator



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## ABSTRACT

The delayed feedback loops play a crucial role in the stability of dynamical systems. The effect of process delay in feedback is studied numerically and theoretically in the delayed feedback nonlinear systems including the neural model, periodic system and chaotic oscillator. The process delay is of key importance in determining the evolution of systems, and the rich dynamical phenomena are observed. By introducing a process delay, we find that it can induce bursting electric activities in the neural model. We demonstrate that this novel regime of amplitude death also exists in the parameter space of feedback strength and process delay for the periodic system and chaotic oscillator. Our results extend the effect of process delay in the paper of Zou et al. (2013) where the process delay can eliminate the amplitude death of the coupled nonlinear systems.

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## 1. Introduction

The self-feedback loops, connecting a system to itself, are pervasive and significant in many science and application fields [1–4], especially in the biological system [5–7]. Many systems perceive external input in the form of self-feedback circuits, or loops. One of the well studied examples is the autapse which has been named originally by Van et al. in 1972 [8]. The autapses, formed between a neuron and a branch of its own axon, are relatively widespread and have been found in the various brain areas, including the cerebellum, striatum, hippocampus, and neocortex [9,10]. So far, several studies have also revealed that these autapses could play an important role in brain function, maintaining the precision of action potential firing trains. Bekkers revealed that excitatory autapses contribute to a positive-feedback loop that maintains persistent electrical activity in neurons, and the functional autapses in the cerebral cortex are found [11]. Saada et. al have identified an autapse which underlies a plateau potential causing persistent activity in the B31/B32 neurons of *Aplysia*. The persistent activity is essential to the ability of these neurons to initiate and maintain components of feeding behavior [12]. Bacci et al experimentally observed that firing precision of spike times of neurons was increased in pyramidal neurons by artificial GABAergic autaptic conductances [13], and found that the autaptic activity has significant inhibitory effects on repetitive firing and increase the current threshold for evoking action potentials [14]. Furthermore, the effect of autaptic on neural dynamics is also investigated widely, and many interesting phenomena are observed [15–22].

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The time delayed feedback, a particularly simple and efficient scheme, has a significant effect on the dynamics of nonlinear systems, especially on stabilizing periodic orbits and steady states embedded in chaotic attractors in physics, chemistry, biology, and medicine [23–25]. For example, Many works have shown that time-delayed feedback methods provide a tool to stabilize unstable steady states [26–29]. Ahlborn and Parlitz found that the multiple delayed feedback signals can stabilize plane waves or the trapping spiral waves in the two-dimensional complex Ginzburg–Landau equation [30]. Gaudreault et al found that the delay feedback can modify the natural frequency of the oscillator and the damping coefficient [31]. Yanchuk presented an asymptotic analysis of time-delayed feedback control of steady states for large delay time, and elaborated analytical conditions for successful control of a fixed point of focus type [32]. The feedback method has also been applied successfully in many experiments, including laser [33], electronic [34], chemical [35], and plasma systems [36].

The time-delayed feedback, used the difference  $g(x(t - \tau)) - g(x(t))$ , was first introduced by Pyragas [37]. Here, the delay  $\tau$  is known as the time delay which arises from finite propagation speeds. It has been named as propagation delay which has also been widely investigated, and various phenomena have been uncovered in time-delayed coupling oscillators [38–45]. Very recently, the marked effects of the process delay on dynamical behavior have been revealed [46–49]. The process delay, different with the propagation delay, comes from a finite response time required for internal processing of the input information. The process delay is universal in a heterogeneous network especially in networks with large hubs due to nodes with different time scale of oscillations. In cognitive psychophysiology and cognitive neuroscience, the mental chronometry is a core paradigm of process delay which the neural system deal with the reaction time to a stimulus [50]. Importantly, the experimental evidence of process delay in annihilating the quenching of oscillation will be reported [51]. So far, to the best of our knowledge, in all existing works on the feedback, the effect of process delay on the dynamical behavior of nonlinear systems was not considered.

In this work, we investigate numerically and theoretically the effect of process delay in self-feedback on dynamics of the nonlinear systems including the neural model, periodic system and chaotic oscillator. We show that the rich dynamical phenomena are observed with the different values of feedback strength, propagation delay, and process delay. For the delayed feedback excitable FitzHugh–Nagumo neural model, the propagation delay can induce simply spiking without process delay. Surprisingly, we find that the process delay can drive the neuron to bursting electric activities. For the periodic system and chaotic oscillator, We find that the process delay can stabilize unstable fixed points before the transmission delay takes no effect in stabilization, and the analytical boundaries are also derived explicitly. This paper is organized as follows. In Section 2, we investigate the FitzHugh–Nagumo neuronal model with delay feedback. In Section 3, the case of the Stuart–Landau system is discussed in details. In Section 4, we examine the dynamical behavior of the delayed feedback Rössler oscillator. Finally, Section 5 is devoted to our conclusions and discussions.

## 2. The FitzHugh–Nagumo neuronal Model

First let us consider the delayed-feedback FitzHugh–Nagumo neuronal model which is described by

$$\epsilon \dot{x} = x - \frac{x^3}{3} - y + I_{syn}, \quad (1a)$$

$$\dot{y} = x + a. \quad (1b)$$

In a neural context,  $x$  is the activator variable (representing the membrane potential) and  $y$  is the inhibitor (related to the conductivity of the potassium channels existing in the neuron membrane) [52–55]. The dynamics of  $y$  is much faster than that of  $x$  because of the small parameter  $\epsilon = 0.01$ . When  $|a| < 1$ , the unit is in the oscillatory regime, while for  $|a| > 1$ , it is in the excitable one [54,55]. In our paper,  $a = 1.05$  is chosen and fixed [54,56,57]. The decision about choosing parameter  $a$  since  $a = 1.05$  is near bifurcation point. The more richer dynamical behaviors may be observed for this chosen parameter.  $I_{syn}$  is the synaptic current through self-feedback,

$$I_{syn} = \kappa (x(t - \tau - \delta) - x(t - \delta)), \quad (2)$$

where the delays  $\delta$  and  $\tau$  physically account for the process time and propagation time, respectively.  $\kappa$  quantifies gain of feedback.

For an electrical autapse with the process delay, the rich firing patterns for the FitzHugh–Nagumo neuronal model are observed. The time series of the action potential for the different autaptic parameters are shown in Fig. 1. In Fig. 1(a) and (b), the autaptic intensity is given as  $\kappa = 0.2$ , and the process delay time is given as  $\delta = 0.8$ . Fig. 1(c) and (d), the autaptic intensity is given as  $\kappa = 0.4$ , and the process delay time is given as  $\delta = 0.2$ . The FitzHugh–Nagumo neuronal model is on quiescent state for this parameter settings, but it transitions to the various spiking patterns in the presence of an process delay. Fig. 1(a) exhibits a period-1 spiking pattern, Fig. 1(b) give a period-2 spiking pattern, while Fig. 1(c) and (d) are devoted to examples of period-4 and period-7 bursting pattern.

To display clearly the effect of process delay in the autapse on the firing patterns, we show the bifurcations of the interspike intervals (ISIs) against propagation delay  $\tau$  with different process delay  $\delta$  and the gain of feedback  $\kappa$ , respectively. The neuronal model is in the periodic oscillatory mode without process delay for the sufficiently large  $\tau$  [Fig. 2(a)]. Comparing three subfigures, the marked effect of the process delay in autaptic is observed [Fig. 2(b) and (c)]. The neuron transmits from

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