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Formulation of a Cooperative-Confinement-Escape problem of multiple cooperative defenders against an evader escaping from a circular region

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ABSTRACT

In this paper, we propose and formulate the *Cooperative-Confinement-Escape (CCE)* problem of multiple cooperative defenders against an evader escaping from a circular region, in which the defenders are moving on the circle with attempt to prevent possible escape of a single evader who is initially located inside the circle. The main contributions are summarized as follows: (1) we first provide an effective formulation of the CCE problem, which is an emphasis of this paper, with design of two nonlinear control strategies for the cooperative defenders and the adversarial evader, respectively. Particularly, we consider to include a proper interaction between each pair of the nearest-neighbor defenders, and an adaptive trajectory prediction mechanism in the strategies of the defenders to increase the chance of successful confinement. (2) For the first attempt on analyzing the CCE dynamics which is unavoidably strongly nonlinear, we analyze the minimum energy of the evader for possible escape. (3) For understanding of the behaviors of the system under different parameters, (i) we illustrate the effectiveness of the confinement strategy using the adaptive trajectory prediction mechanism, and (ii) the physical roles of the system parameters with respect to the system dynamics, some of which may be unexpected or not straightforward. A separate paper will be presented for systematic analysis of the agents' behaviors with respect to the large intervals of the parameter settings.

1. Introduction

There are many interests in pursuit–evasion and predator–prey problems in multiple disciplines, as in ecology and biology [1–10], mathematics [24,27–29,31–37,40], physics [11,13,16], computer science [31–36,46], and control and robotics [22,25,26,29–33,38,41–44]. For example, the defender–intruder problem and the optimal interception strategy [12], the Homicidal Chauffeur problem [22,26], the *princess and monster game* or the lion and man problem [27], and the confinement-escape problem of a defender against an evader escaping from a circular region [14], with escape analysis of the confinement-escape problem provided in [15], etc. Typically, a pursuit–evasion problem has two adversarial agents, i.e., there is generally one agent against another, with a fewer cases of multiple agents [7,11,13,16,25,39,41], or using wireless sensor networks [43]. For predator–prey problems, most research is in the field of biology, with main concerns on variation

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of the populations of the predators and preys as the system evolves. For research on pursuit–evasion, one category of the main concerns focuses on search algorithms for a graph (i.e., how the pursuers search the vertices of the graph effectively to detect the evader) [31–36], typically with no explicit kinematics or dynamics of robotic agents considered (refer to the review paper [31]). There are some work on robotic pursuit–evasion from the perspective of control [25,29,41].

Different from the above scenarios of the pursuit–evasion problems, in this paper we propose the problem of multiple defenders patrolling on a predefined region against escape of an evader who is initially located inside the region, which is referred to as the *Cooperative-Confinement-Escape* problem, or for abbreviation, the CCE problem. When there is only one defender in the CCE problem, we call it the *Confinement-Escape* problem [14], or for abbreviation, the CEP. Cooperation between the defenders in the proposed CCE problem is a newly added feature and also one of the components in the strategies of the defenders, as compared with the CEP; note that here cooperation in the CCE problem is still different from the cooperation interactions in, e.g., collective motion in physics [11,13,16–18] or cooperative control of multiple agents (such as flocking [19], formation [19,20,42], cooperative manipulation [21], swarming on non-Euclidean manifold [20,23]), etc.

In this paper, we consider the CCE problem particularly with respect to a circular region in the 2-D Euclidean space, i.e., multiple defenders moving on the circle for boundary patrolling with attempt to prevent escape of the evader who is initially located inside the circle. The defenders try to guard the circle with cooperation to prevent possible escape of the evader, while the evader attempts to escape from the circular region while avoiding the defenders. As a result, successful confinement is defined that, the evader will be always confined inside the circle; otherwise, successful escape of the evader is defined.

First, we focus on modeling of the system evolution with an emphasis on design of the strategies for the defenders and the evader, respectively. Here, we are not intended to formulate the problem as a differential game [22]; rather, we consider the modeling of the problem using possibly bio-inspired strategies of the agents which are simple yet effective enough with rich nonlinear behaviors of the agents; this modeling is itself meaningful and will be a good start point for considering more complex behaviors (including gaming behaviors) of the agents in applications. The constraint of spherical or circular motion of agents was provided in the author's previous work [14,20].

There are three fundamental ingredients in the design of the strategies of the agents, i.e., (1) modeling of the agents, (2) design of interactions for cooperation between the defenders and for conflict between the defender and the evader, and (3) the control laws of the agents. First, for simplicity at the initial stage, we model every agent as a mass point in the Lagrangian approach with second-order dynamics that is described by ordinary differential equations (ODEs). Second, to model interactions between the agents, we assume the fundamental interactions (i.e., only attractions and repulsions) between the agents for the CCE evolution, to be specific: (i) from the perspective of every defender, there is an attraction on the defender from the evader while not driving the defender out of the circle [14], (ii) from the perspective of the evader, there is a direct repulsion on the evader from each of the defenders, and (iii) for the purpose of cooperative deployment on a large-scale region on the circle, assume that there is a proper repulsion between *each pair of the nearest-neighbor defenders*. The last ingredient is to design the control laws of the agents with some possible adaptive mechanisms that favor successful confinement.

Integrated with the above considerations, we design two nonlinear control strategies for the defenders and the evader, respectively, with consideration of including an adaptive trajectory prediction mechanism in the strategy of the defenders to increase the chance of successful confinement.

The main contributions in this paper are summarized in three aspects: First, as an emphasis of this paper, we provide an effective formulation of the CCE problem, with design of two control strategies for the defenders and the adversarial evader, respectively. Particularly, we consider to include a proper repulsion interaction between each pair of the nearest-neighbor defenders, and an additional trajectory prediction mechanism in the strategies of the defenders to increase the chance of successful confinement. Second, for the first attempt on analyzing the CCE dynamics which is unavoidably strongly nonlinear, we analyze the minimum energy of the evader for possible escape. Third, for understanding of the behaviors of the system under different parameters, which are mostly illustrative with examples, (i) we illustrate the effectiveness of the confinement strategy using the adaptive trajectory prediction mechanism, and (ii) the physical roles of the system parameters with respect to the system dynamics, some of which may be unexpected or not straightforward, and (iii) we investigate the dynamic properties of the system, using some kinds of performance indices.

Finally, for systematic analysis of the behaviors of the agents with respect to the larger intervals of the parameter settings, a separate paper will be soon presented on this concern, together with the winning sets for the defenders and the evader, respectively.

This remaining of the paper is arranged as follows: Section 2 describe the CCE problem of multiple defenders against escape of an evader. Section 3 designs the components of the control laws of the defenders and evader, respectively. Section 4 provides the overall control laws of the agents. Section 5 analyzes minimum energy of the evader for possible escape. Section 6 analyzes the dynamics and performance of the system. Section 7 discuss motion patterns of the system with different number of the defenders and different parameters of the system. Section 8 is future considerations. Section 9 is the conclusion.