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A rescaling algorithm for the numerical solution to the porous medium equation in a two-component domain

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ABSTRACT

The aim of this paper is to design a rescaling algorithm for the numerical solution to the system of two porous medium equations defined on two different components of the real line, that are connected by the nonlinear contact condition. The algorithm is based on the self-similarity of solutions on different scales and it presents a space-time adaptable method producing more exact numerical solution in the area of the interface between the components, whereas the number of grid points stays fixed.

1. Introduction

Let $0 < m, \sigma < 1$ be given. We present a numerical study of the model problem

$$(u^m)_t - u_{xx} = 0 \quad -\ell < x < 0, \quad 0 < t < T, \tag{1}$$

$$(v^\sigma)_t - v_{xx} = 0 \quad 0 < x < \ell, \quad 0 < t < T \tag{2}$$

for given $\ell > 0$, where nonnegative $u = u(x,t)$, $v = v(x,t)$ satisfy the following contact conditions for $x = 0$

$$u_x(0,t) = v_x(0,t) \quad 0 < t < T, \tag{3}$$

$$v(0,t) = M u^\omega(0,t) \quad 0 < t < T \tag{4}$$

for given $0 < M, \omega < \infty$, and boundary and initial conditions, respectively,

$$u(-\ell,t) = v(\ell,t) = 0 \quad 0 < t < T, \tag{5}$$

$$u(x,0) = u_0(x) \quad -\ell < x \le 0\,, \tag{6}$$

$$v(x,0) = 0 \quad 0 < x < \ell, \tag{7}$$

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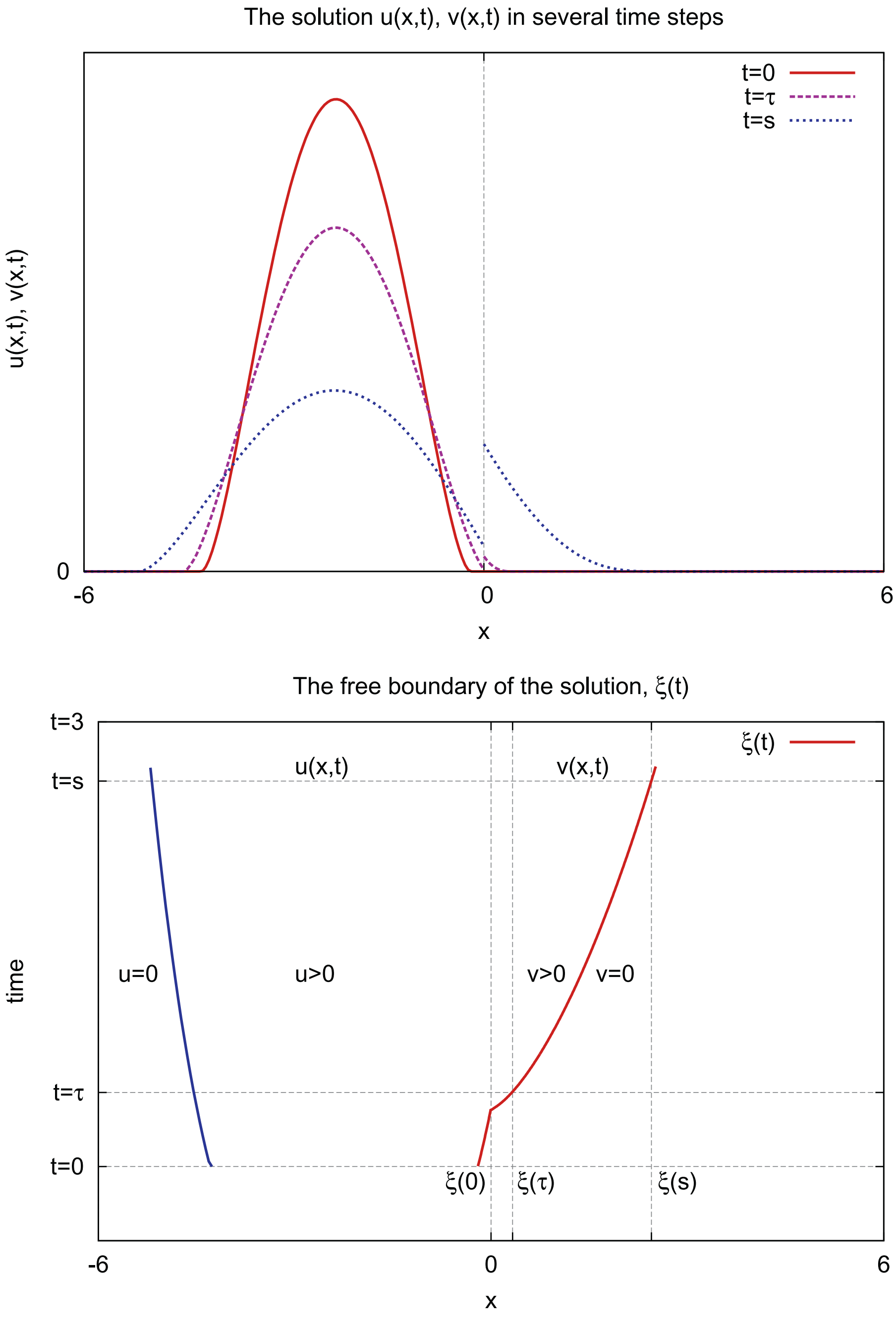

Fig. 1. Numerical solution of the Problem (1)–(7) for $m = 0.4$, $\sigma = 0.7$, $M = 2.2$ and $\omega = 1.4/1.7$.

where u_0 is the initial function of Barenblatt's solution located inside the interval $(-\ell, 0)$, see [4,13]. For reader convenience, let us sketch the corresponding solution of Problem (1)–(7) for $m = 0.4$, $\sigma = 0.7$, $M = 2.2$ and $\omega = 1.4/1.7$ in some time steps $0 < \tau < s < T$, see Fig. 1, where

$$\xi(t) = \begin{cases} \sup\{x \in [-\ell, 0] : \ u(x,t) > 0 \text{ and } [v(x,t) \equiv 0 \ \forall x \in [0,\ell]]\} \\ \sup\{x \in [0,\ell] : \ v(x,t) > 0\} \end{cases}.$$

We shall call the set $\{(0, t), 0 < t < T\}$, that separates two various components, the interface and the function $\xi = \xi(t)$ for $t \in [0, T]$ we shall address as the free boundary of our problem.

The motivations of our study are two-fold. First, system (1)–(7) describes qualitative behaviour of a free boundary in diffusion processes that arise by the modelling of dermal and transdermal drug delivery. Especially, its solution