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Short communication

A new material identification pattern for the fractional Kelvin–Zener model describing biomaterials and human tissues

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ABSTRACT

The aim of this study is to describe several biomaterials and tissues using a simple material identification pattern applied to the fractional Kelvin–Zener model of viscoelastic body and standard mechanical tests. Each of the descriptions comprises the order of fractional derivative of stress and strain, modulus of elasticity, and stress and strain relaxation constants that obey restrictions imposed by the Clausius–Duhem inequality. These four parameters are obtained by use of the Laplace transform, Post's inversion formula and Newton's method. The suggested approach can serve as an alternative to quasilinear viscoelasticity providing a physically uniform quantitative measure for biomaterials/tissues comparison and can be applied to real data. It works for nonsmooth inputs too. Regarding biomaterials the comparison between an etched poly lactic-co-glycolic acid membrane and the corresponding composite scaffold was made. With respect to human tissues the tympanic membrane, the stapedial tendon, and the stapedial annular ligament were described. The obtained mechanical response for examined cases is in agreement with the experimentally recorded one.

1. Introduction

In the last decades, increasing number of papers on applications of fractional calculus (FC) in biology and medicine is noticeable [1,6-8,21-33]. The notion of fractional differentiation, or more appropriately the differentiation of arbitrary real order, means an operation analogous to standard differentiation which will take into account, memory effects if the independent variable is time, or nonlocal effects in the case of spatial independent variables. Being a part of a constitutive law, [20], this more sophisticated tool is naturally used to complement fundamental principles and contributes to the reliability of any simulation of organ deformations under physiologic loads. Namely, for any material that is to be used in biomedical engineering, considerations of mechanical properties are essential. A good choice of material for a particular application depends on recognition of these properties in the analysis of both similarity of biomaterials to biological ones and bio-compatibility since mechanical functions should be treated along the fundamental principles of physiology, cell biology and basic clinical knowledge. It is clear that even in a case of typical load-deformation patterns, the characterization of mechanical response requests more sophisticated stress–strain relations and efficient material identification procedures.

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Among all possible constitutive laws for complex applications in bioengineering we recommend the fractional Kelvin–Zener model (FKZM) that generalizes the standard linear solid known as Kelvin's [2], or Zener's model, [3]. Besides fractional derivatives of stress and strain it calls for the restrictions that follow from second law of thermodynamics, [4,5], so it takes into account energy dissipation ab initio. Comparing to the quasi-linear viscoelasticity model of Fung, the most common phenomenological model of viscoelastic behavior that is based on Prony's approximation, FKZM requires only four constants for acceptable accuracy. Regarding the material identification procedures for FKZM, these constants are usually calculated from highly non-linear systems involving Mittag–Leffler-type functions combined with valid iterative methods which will enable an initial guess of material parameters to be systematically improved. Examples are given in [6] and [7] where Newton's method and the particle swarm optimization (PSO) were used respectively.

This work is motivated by the need to avoid if-then-else statements usually encountered in the computational schemes included in the evaluation of the Mittag–Leffler function. Namely, by analogy with a new fractional system identification procedure recently proposed in [8] that bears on the Laplace transform, Post's inversion formula (PIF), Newton's method and the computational software program support, offered by Mathematica, instead of the exact solutions of fractional differential equations their analytical approximations will be used. It will be shown that the suggested approach combined with FKZM fits in with standard experimental patterns and yields acceptable accuracy even in cases with nonsmooth inputs, i.e. when prescribed load or deformation functions exhibit corner points. Typical examples of these nonsmooth patterns are ramp-and-hold stress then creep and ramp-and-hold strain then stress relaxation experiments. Without loss of generality and in order to show how it works in practice, recently reported results on ramp-and-hold strain stress relaxation (RHSSR) experiments will be used. With respect to biomaterials, on the basis of [9], the comparison between an etched poly lactic-co-glycolic acid (PLGA) membrane and the corresponding composite scaffold will be made. In the same manner, FKZM representation of the tympanic membrane (TM), the stapedial tendon (ST), and the stapedial annular ligament (SAL) will be obtained by use of reports given in [10,11] and [12], respectively. Assuming that similar tissues can be recognized by comparison of their FKZM representations these results can be used in finding tissue analogues needed for reconstructions of the middle ear function [7].

2. Theory/calculation

Consider uniaxial isothermal deformation of a viscoelastic body of negligible mass from its virginal state. Denoting strain (relative elongation) by ε and stress (force per unit area of the body in the undeformed state) by σ, let us analyze the deformation pattern denoted by RHSSR. Namely, in the loading phase with initial constant strain rate κ, at time instant t_k strain will reach a constant level $\varepsilon_0 = \kappa t_k$ and stress will attain its maximal value, say σ_k. After t_k, strain is maintained constant while stress starts to decrease in time without any jump at t_k. This stress relaxation is an inherent property of real materials. The strain function corresponding to RHSSR reads $\varepsilon(t) = \kappa[t \cdot h(t) - (t - t_k)\,h(t - t_k)]$, where $h(t)$ stands for the Heaviside step function, while κ, and either ε_0 or $t_k$are input parameters. Note that there is a corner point of $\varepsilon(t)$ at $t = t_k$.

It is a well known fact that RHSSR deformation pattern can be easily described by FKZM, see [7], i.e.

$$\sigma + \tau_{\sigma\alpha}\sigma^{(\alpha)} = E_\alpha(\varepsilon + \tau_{\varepsilon\alpha})\varepsilon^{(\alpha)}, \tag{1}$$

where $(\cdot)^{(\alpha)}$ denotes the left Riemann–Liouville fractional derivative of order $\alpha$with respect to time, taken in the standard form [13], E_α is modulus of elasticity, while $\tau_{\sigma\alpha}$ and $\tau_{\varepsilon\alpha}$ are the relaxation constants of dimension time to the power of α, $(0 < \alpha \leq 1)$. The corresponding fundamental restrictions on the coefficients in (1), following from the Clausius–Duhem inequality read $E_\alpha > 0,\ \tau_{\sigma\alpha} > 0,\ \tau_{\varepsilon\alpha} > \tau_{\sigma\alpha}$, see [4] and [5]. As stated in [14] the same analysis can be applied for the simple shear deformation pattern, where in turn ε, σ, and E_α in (1) will denote shear strain, shear stress, and shear modulus respectively.

Applying Laplace's transform with $\bar{\sigma} = \bar{\sigma}(s) = L\{\sigma(t)\} = \int_0^\infty e^{-st}\sigma(t)\,dt$ and $\bar{\varepsilon} = \bar{\varepsilon}(s) = L\{\varepsilon(t)\} = \int_0^\infty e^{-st}\varepsilon(t)\,dt$, to (1), under the assumption that the deformation process starts from a virginal state $\varepsilon(0) = 0,\ \sigma(0) = 0$, after some calculations one gets

$$\bar{\sigma}(s) = E_\alpha\bar{\varepsilon}(s) + E_\alpha(\tau_{\varepsilon\alpha} - \tau_{\sigma\alpha})\frac{1}{s^{-\alpha} + \tau_{\sigma\alpha}}\bar{\varepsilon}(s), \tag{2}$$

where the standard expression for the Laplace transform of the fractional derivative of order α was used [13]. The Laplace transform of the strain function corresponding to RHSSR reads $\bar{\varepsilon}(s) = \kappa(1 - e^{-st_k})/s^2$. It should be noted that for RHSSR the exact analytical solutions of (2) involving the Mittag–Leffler type functions are used in material identifications procedures in time domain, see [7] and [8] for details. Working in time domain has its own advantages [15], however the Mittag–Leffler type functions appearing within iterative processes of these procedures, either directly, or as kernels of convolution integrals, are calculated. Depending on the argument these calculations bears on several expressions and if-then-else statements. In the following these steps will be replaced by use of PIF. This formula approximates the original of the Laplace transform in a given point in time domain, see [16] for proof and [17] for its application within a fractional evolution problem of short duration.

In order to determine FKZM representation, i.e., the values of α, E_α, $\tau_{\sigma\alpha}$, $\tau_{\varepsilon\alpha}$, first four non-equidistant time instants within four pairs from the set of experimental data obtained for given $\varepsilon(t)$, say $\{(t_j, \sigma_j),\ j = 1, 4\}$ are to be randomly selected. Then, introducing the corresponding strain image $\bar{\varepsilon}(s)$ in (2), and regarding obtained $\bar{\sigma}(s)$ as a function