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Interplay between detection strategies and stochastic resonance properties

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ABSTRACT

We discuss how to exploit stochastic resonance with the methods of statistical theory of decisions. To do so, we evaluate two detection strategies: escape time analysis and strobing. For a standard quartic bistable system with a periodic drive and disturbed by noise, we show that the detection strategies and the physics of the double well are connected, inasmuch as one (the strobing strategy) is based on synchronization, while the other (escape time analysis) is determined by the possibility to accumulate energy in the oscillations. The analysis of the escape times best performs at the frequency of the geometric resonance, while strobing shows a peak of the performances at a special noise level predicted by the stochastic resonance theory. We surmise that the detection properties of the quartic potential are generic for overdamped and underdamped systems, in that the physical nature of resonance decides the competition (in terms of performances) between different detection strategies.

1. Introduction

It is well known that stochastic resonance (SR) can be exploited, under suitable circumstances, to improve detection [1–4]. However, the knowledge that the system response could possibly be enhanced at special noise level is not sufficient to identify the best method to be employed for signal detection. In fact, several indexes have been proposed to quantify the response of a system when SR occurs: e.g., Fourier spectrum analysis [1,2], synchronization measures through strobed data [4], amplification of a reaction coordinate [5], escape times processing [6–8]. Therefore, it would be interesting to know in advance which method will actually work better. Put it in another way, it is interesting to know if a link exists between the physics characteristics of the system and the performance behavior of feasible detection strategies.

Let us recall some basic concepts of SR from the point of view of statistical decision theory. To be specific, we consider a signal $S(t)$ that is a mixture of a periodic drive of amplitude α, (angular) frequency ω, initial (unknown) phase φ_0, and a random (uncorrelated Gaussian) perturbations ξ:

$$S(t) = \alpha \sin(\omega t + \varphi_0) + \xi(t) \quad (1)$$

The conventional wisdom is that SR occurs when the response of a nonlinear system to the signal $S(t)$ can be enhanced at a *special* noise level. In the standard analysis of SR [2,3,8], the response is evidenced by the behavior of a

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component of the outgoing Fourier spectrum, that is maximized at a certain noise intensity. The upsurge of the Fourier component makes SR appealing for signal detection, inasmuch as it is conceivable to exploit the Fourier analysis to reveal the presence of an injected deterministic signal [9]. In this scheme, one hopes that a particular combination of noise and periodic drive makes it easier to detect the signal, for instance because a threshold is only passed with the help of the noise [10] (although it has been proposed to exploit stochastic resonance also for suprathreshold deterministic signals [11]). However, the very concept that noise could be beneficial is counterintuitive, if not controversial [12,13], in that it sounds against good sense that *more* disturbances can, in the end, improve the detection of a signal. This Gordian knot has been cut by noticing that SR only helps detection in *suboptimal* systems [14,15], and suboptimal threshold choices [16]. In less general terms, let us suppose we want to decide about the presence of a periodic drive; then decision theory proves that the Neyman–Pearson scheme is optimized by the likelihood ratio test (LRT). To decide about the presence of a periodic forcing corrupted by white Gaussian noise LRT amounts to the scalar product of the signal $S(t)$ and the mask, the periodic component $\sin(\omega t)$. This is the optimal detection strategy – the matched filter applied to the input signal $S(t)$ [17] – that cannot be improved adding noise. In practice, one is often frustrated in the application of LRT optimal technique and employs suboptimal strategies where SR based signal detection enhancement can genuinely occur. As preeminent examples where the optimal strategy is not feasible, we can mention the cases where the full trajectory $S(t)$ is difficult to retrieve (as in very sensitive Fabry–Perot pendulums for gravitational waves detection [18,19]), or it is just not available for measurements (as in Josephson junctions, for the quantum mechanical nature of the dynamical variable [20–22]). In some other cases [23] the recorded signal is far too long to be analyzed with optimal methods, and the LRT is de facto not applicable. In still other cases a nonlinearity transformation in the system allows for the occurrence of typical SR pattern [16]. To visualize the difficulty, we can imagine to tackle the original problem where SR arose: the study of climate changes, based on geological evidence of the alternate of dry and cold periods [24]. Can we access the instantaneous temperature of the Earth, that is the signal $S(t)$? Unfortunately, we can just estimate the passage from an ice age to a dry age, i.e. the escape time from stable climate configurations. Even if better techniques were available to estimate the yearly Earth temperature, say from the maximum extension of continental ice sheets, this amounts to measure each year the temperature in the coldest day. Put another way, one could only observe the so-called strobed dynamics obtained illuminating the system at some time intervals. The two sampled dynamics we have just mentioned – escape times (ET) from dry to cold periods and strobed dynamics (SD) at prescribed time intervals – are suboptimal, in that the optimal LRT strategy, equivalent to the matched filter, requires to exploit the whole trajectory (i.e., the full information content, or the instantaneous temperature) of the input signal. Only for the suboptimal strategies, based on the reduced data (e.g. the escape times or the strobed dynamics), SR can occur. Indeed it has been shown that: (i) noise can be used to enhance signal detection through the analysis of ETs in the first order standard bistable potential [8] and in a second order washboard potential [25] (ii) with the appropriated choice of noise intensity SD exhibits good detection performances, ≈ 3 dB below the optimum, when strobing occurs at the forcing period $2\pi/\omega$ [4].

Thus the alternatives to the matched filter are practically attractive, and could possibly exhibit a *bona fide* enhancement when noise increases. If one backs down the optimal matched filter, physical intuition suggests to seek for best performances in the parameter region where SR occurs. Put it another way, if an opportunity to improve the analysis by adding noise is to exist, one guesses that the resonant condition of SR is the first place where to look for such a chance.

Still, an open problem can be posed: which technique, among the many suboptimal ones, best performs in a specific physical system? The objective of the present work is to systematically characterize the two detection performances of two reduced data – the escape times and the strobed dynamics – for the overdamped and underdamped prototypal system of a quartic potential. The analysis of the performances demonstrates that the two techniques display different properties, and that the best performing strategy depends upon the underlying dynamical nature of the resonance. We remark that we have selected two particular strategies which are suitable when the full trajectory is not easily available. Other suboptimalities can be devised, for example threshold detectors, similar to a neuron, that are most suited in biological applications [16,26].

The paper is organized as follows: in Section 2 we outline the model equations. In Section 3 we establish the suboptimal detection strategies for ET and SD, that are applied and evaluated for the prototypal quartic potential in Section 4. Section 5 concludes with the physical consequences of the above analysis.

2. Models for bistable systems

2.1. First order model

Let us consider the signal (1) applied to a prototypal quartic bistable potential; in normalized units the system is governed by the following stochastic differential equation [3]:

$$\frac{dx}{dt} - x + x^3 = \alpha \sin(\omega t + \varphi_0) + \xi(t) = S(t). \tag{2}$$

Eq. (2) is called overdamped because it is the high friction limit of a nonlinear oscillator driven by a mixture of deterministic signal and noise, as will be discussed in the next subsection. In Eq. (2) a random term appears, $\xi(t)$, to model an additive noise with autocorrelation function of intensity D, viz. $\langle \xi(t)\xi(t')\rangle = 2D\delta(t-t')$, corrupting the external sinusoidal drive. Thermal noise has been supposed uncorrelated with external noise and included in the overall noise D. For practical applications the physical nature of the noise sources (either intrinsic thermal noise or external noise) does affect the mathematical treatment of the detection strategies (if the sources are uncorrelated). We focus on the simple case of additive noise (that is a paradigm