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Weighted multifractal cross-correlation analysis based on Shannon entropy

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ABSTRACT

In this paper, we propose a modification of multifractal cross-correlation analysis based on statistical moments (MFSMXA) method, called weighted MFSMXA method based on Shannon entropy (W-MFSMXA), to investigate cross-correlations and cross-multifractality between time series. Robustness of this method is verified by numerical experiments with both artificial and stock returns series. Results show that the proposed W-MFSMXA method not only keep the multifractal structure unchanged, but contains more significant information of series compared to the previous MFSMXA method. Furthermore, analytic formulas of the binomial multifractal model are generated for W-MFSMXA. Theoretical analysis and finite-size effect test demonstrate that W-MFSMXA slightly outperforms MFSMXA for relatively shorter series. We further generate the scaling exponent ratio to describe the relation of two methods, whose profile is found approximating a centrosymmetric hyperbola. Cross-multifractality is found in returns series but then destroyed after being shuffled as a consequence of the removed long memory in separate series.

1. Introduction

Sequences in complex systems generally exhibit fluctuations on a wide range of time scales of the data, where the distinct fluctuations follow a scaling relation over several orders of magnitude. Such scaling laws provide a description of the data and the complex system by fractal scaling exponents. Nevertheless, many records do not present a simple monofractal scaling behavior and different parts of the series require different scaling exponents. In even more complicated cases, such different scaling behavior can be observed for many interwoven fractal subsets of the time series, where a multitude of scaling exponents is required for a full characterization of the scaling behavior in the same range of time scales and a multifractal analysis must be applied. In order to observe the fractals and multifractals in time series, algorithms have been developed one by another [1,2].

Since detrended fluctuation analysis (DFA) has been proposed by Peng et al. [3] to detect the long-range power-law correlations in DNA sequences, it has been successfully applied to diverse fields [4–11]. Then Podobnik and Stanley [12] generalized DFA and introduced the detrended cross-correlation analysis (DCCA) for two non-stationary time series, which has aroused increasing interest in analysis of long-range cross-correlation and multifractality [1,13–25]. Specifically, the analysis is based on the bivariate Hurst exponent h_{xy} estimation, which is related to an asymptotic power-law decay of the cross-correlation function. A power-law cross-correlated process has the cross-correlation function $C_{xy}(k) \propto k^{2h_{xy}-2}$ for $k \to +\infty$ and the cross-power spectrum $|f_{xy}(s)| \propto s^{1-2h_{xy}}$ for $s \to 0+$. $h_{xy} = 0.5$ is characteristic for the absence of power-law cross-correlation, while processes with

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$h_{xy} > 0.5$ are cross-persistent and $h_{xy} < 0.5$ indicates the anti-persistent cross-correlation of the data [2,12,25]. Basis of these, the cross-correlation coefficient (σ_{DCCA}) was introduced with the objective of quantifying the level of cross-correlation between non-stationary time series [26–28]. In the DFA–DCCA-σ_{DCCA} line, Kristoufek [29] has recently introduced the DFA framework to estimate regression parameters at different scales and under potential non-stationarity and power-law correlations. Alternatively, the detrending moving-average (DMA) method can also be used for fractal or multifractal analysis, found that the backward multifractal detrending moving average (MFDMA) algorithm outperforms the multifractal detrended fluctuation analysis (MFDFA) [30–33]. As an alternative to σ_{DCCA}, based on the detrending moving-average cross-correlation analysis (DMCA), the DMCA coefficient ρ_{DMCA} was proposed as well [17,34]. To investigate the multifractal features of two cross-correlated non-stationary series, several methods have been proposed for this purpose. Zhou [14] further proposed multifractal detrended cross-correlation analysis (MFDCCA) as a generation of DCCA. Jiang and Zhou [35] developed a class of MFDCCA algorithms based on DMA, called MFXDMA whose performances are comparative to MFDCCA's. Moreover, a new method for the detection of long-range cross-correlations and multifractality based on scaling of qth order covariances, multifractal height cross-correlation analysis (MF-HXA), was proposed by Kristoufek [18]. Meanwhile, statistical moments function or partition function has been widely applied to analyze the multifractal features of financial time series [36–40]. Based on this approach, Jiang and Zhou [38] investigated the multifractal characteristics in intraday minutely data for four stock market indexes within individual trading days and found the so-called multifractality is merely an illusion. Dissimilarly, when they performed the partition function approach on the 1-min volatility of two indexes and 1139 stocks in the Chinese stock markets, multifractal nature is significant [39]. Extending that from one dimension to two dimensions, Wang et al. [40] introduced the multifractal cross-correlation analysis based on statistical moments (MFSMXA) as an application of the joint multifractal measures [41], finding that MFSMXA has comparative performance with MFXDMA and sometimes perform slightly better than MFDCCA. By far, multifractal analysis has been employed successfully in various provinces, such as human nature [42–44], financial time series [45–47], river flow [48] and traffic signals [49], etc.

However, there are limitations in these algorithms and the common one is that they ignored the difference of the measurement in each segment that contributes to the final fluctuation functions unequally theoretically. While in other academic fields, weighting function is selected for improvement, such as Shannon entropy [50–53], variance or energy [54], and so on [55]. Inspired by their work, basis of MFSMXA, we propose in this paper a weighted method based on Shannon entropy (W-MFSMXA) to analyze the multifractal cross-correlation between two series. Results show that W-MFSMXA is comparable to and in some extent better than MFSMXA.

The remainder of this paper is organized as follows. In Section 2, the methodologies of MFSMXA and W-MFSMXA are introduced. In Section 3, we test the effectiveness of the W-MFSMXA algorithms with three types of artificial time series: two-component ARFIMA stochastic processes, binomial multifractal model and the NBVP time series. Performances are also compared with these in MFSMXA. Application to financial time series is presented in Section 4. At last, Section 5 gives the conclusions and appendix exhibits charts of the finite-size effect for the binomial multifractal model.

2. Methodologies

2.1. MFSMXA method

The MFSMXA method [40] consists of four steps. Consider two time series {X(i)} and {Y(i)} of the same length N, where $i = 1, 2, \ldots, N$.

Step 1: Divide each series into $N_s = \mathrm{int}(N/s)$ non-overlapping segments of equal length s. For a given segment size s, the elements in the vth segment can be described as:

$$x(i,s) = x((v-1)s+i) \quad \textit{and} \quad y(i,s) = y((v-1)s+i), \tag{1}$$

where $v = 1, 2, \ldots, N_s$ and $i = 1, 2, \ldots, s$.

Step 2: For each segment, we define a quantity u as follows,

$$\begin{aligned} u_X &= u_X[(v-1)s+1, vs] = \sum_i x[(v-1)s+i] \\ u_Y &= u_Y[(v-1)s+1, vs] = \sum_i y[(v-1)s+i], \end{aligned} \tag{2}$$

where $[(v-1)s+1, vs]$ is the vth segment. The measure μ in each segment is constructed as follows,

$$\begin{aligned} \mu_X(v,s) &= u_X(v,s) \Bigg/ \left(\sum_{v=1}^{N_s} u_X(v,s) \right), \\ \mu_Y(v,s) &= u_Y(v,s) \Bigg/ \left(\sum_{v=1}^{N_s} u_Y(v,s) \right). \end{aligned} \tag{3}$$