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Conditions for persistence and ergodicity of a stochastic Lotka–Volterra predator–prey model with regime switching

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ABSTRACT

A stochastic Lotka–Volterra predator–prey model with regime switching is investigated. We examine when the system is persistence in mean and the extinction under some appropriate conditions, and we discuss the threshold between them. Sufficient conditions for the stationary distribution which is ergodic and positive recurrent of the solution is established using stochastic Lyapunov functions. Simulations are also carried out to illustrate our analytical results.

1. Introduction

The dynamic relationship between predators and their preys has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology [22]. In addition to mathematics and biology, the application of predator–prey models are also reflected in other areas, for example, physics. Recently, predator–prey interaction have been studied in structured populations in cyclical interactions with alliance-specific heterogeneous invasion rates [34] and in noise-guided evolution within cyclical interactions [35]. Reviews of modern physics pointed out that the potential applicability of the proposed theory extends to statistical mechanics of evolutionary and coevolutionary games, in collective behavior and evolutionary games [36], see [37,38] for more related theories. We focus on the developments and changes of predator–prey models, a vast wealth of literature has investigated on them. From the early simple predator–prey models to the later Gauss-type [29], Leslie–Gower model [25], ratio-dependent predator–prey system [23,39], Holling type [24], Beddington–DeAngelis functional response model [26,27], etc., and based on these models, many scholars have studied the non-autonomous and periodic coefficients model, the models with impulsive perturbations [40] or with time delays [30] or on time scales [28]. In addition, taking into account that nature is full of uncertainty and random phenomena, differential equations disturbed by environment white noise, colored noise have been studied [31–33,41], recently, and the existence and uniqueness of solutions, persistence, stationary distribution and ergodicity, global attractivity and extinction of stochastic systems have been discussed [9–17,20].

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From all of the above models, it can be seen that Lotka–Volterra predator–prey models occupy an important position. The classical Lotka–Volterra predator–prey model with white noise is expressed by

$$\begin{cases} dx(t) = x(t)[a_1 - b_1 y(t) - c_1 x(t)]dt + \alpha x(t)dB_1(t), \\ dy(t) = y(t)[-a_2 + b_2 x(t) - c_2 y(t)]dt + \beta y(t)dB_2(t). \end{cases} \tag{1.1}$$

The stochastic processes $x(t)$ and $y(t)$ represent, respectively, the prey and predator population. Now $a_i (i = 1, 2)$ denotes the intrinsic growth rate of the corresponding population, the coefficient b_1 is the capturing rate of the predator, b_2 stands for the rate of conversion of nutrients into the reproduction of the predator, and c_i represents the density-dependent coefficients of the prey and the predator, respectively. Here $B_i(t)$ is one-dimensional standard Brownian motion, α^2 and β^2 stand for the intensities of the white noises on the prey and predator population, respectively. Note all these coefficients are positive constants. Chessa et al. [4] studied the system (1.1) and obtained the existence, uniqueness and non-extinction property of the solution, whereas Rudnicki [2] proved the system (1.1) has a unique distribution density using the theory of Markov semigroups assuming that the random noise $B_1(t)$ and $B_2(t)$ are correlated. System (1.1) was studied extensively (see [1,3,5,6] and the references therein).

Besides environment white noise, in this paper we will also consider another colored noise, say telegraph noise [7,8]. Telegraph noise can be described as a random switching between two or more environmental regimes, which differ in terms of factors such as nutrition or rainfall. Let $\{r(t), t \geq 0\}$ be a right-continuous Markov chain on the probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t \geq 0}, \mathbb{P})$, taking values in a finite-state space $\mathbb{S} = \{1, \ldots, m\}$. The generator $\Xi = (\theta_{ij})_{1 \leq i,j \leq m}$ is given, for $\delta > 0$, by

$$P\{r(t+\delta) = j | r(t) = i\} = \begin{cases} \theta_{ij}\delta + o(\delta), & \text{if } i \neq j, \\ 1 + \theta_{ij}\delta + o(\delta), & \text{if } i = j. \end{cases}$$

Here θ_{ij} is the transition rate from i to j and $\theta_{ij} \geq 0$ if $i \neq j$, while $\theta_{ii} = -\Sigma_{i \neq j}\theta_{ij}$. In this paper we assume $\theta_{ij} > 0$ if $i \neq j$. Suppose that the Markov chain $r(t)$ is independent of the Brownian motion $B(\cdot)$ and it is irreducible. Under this condition, the Markov chain has a unique stationary (probability) distribution $\pi = (\pi_1, \ldots, \pi_m) \in R^{1 \times m}$, which can be determined by solving the linear equation $\pi \Xi = 0$, $\Sigma_{k=1}^m \pi_k = 1$ and $\pi_k > 0$, $\forall k \in \mathbb{S}$. We refer the reader to [9–11] for details.

The population stochastic system (1.1) with regime switching can be described by the following model

$$\begin{cases} dx(t) = x(t)[a_1(r(t)) - b_1(r(t))y(t) - c_1(r(t))x(t)]dt + \alpha(r(t))x(t)dB_1(t), \\ dy(t) = y(t)[-a_2(r(t)) + b_2(r(t))x(t) - c_2(r(t))y(t)]dt + \beta(r(t))y(t)dB_2(t). \end{cases} \tag{1.2}$$

Assume, for any $k \in \mathbb{S}$, that the coefficients $a_1(k), a_2(k), b_1(k), b_2(k), c_1(k), c_2(k), \alpha(k)$ and $\beta(k)$ are positive. Here $B_i(t) (i = 1, 2)$ are independent standard Brownian motions.

The main aim of this paper is to investigate the persistence in mean and the extinction of system (1.2) and find the threshold between them, which is important for assessing the risk of extinction of a population exposed to an environmental toxicant. By constructing suitable Lyapunov functions, we give sufficient conditions for system (1.2) being positive recurrent (and the existence of a unique ergodic stationary distribution). Liu and Wang [12] investigated a logistic model driven by Brownian noise and colored noise and obtained the threshold between weak persistence and extinction. There are only a few results on the model (1.2) and other stochastic predator–prey systems under regime switching. Stationary distribution of stochastic population models were studied in [15–17] and Zhu and Yin [18] studied the ergodicity of a stochastic model with Markov switching. Using the theory in [18], Liu et al. [13] and Settati and Lahrouz [14], investigated a stochastic Lotka–Volterra mutualistic system with regime switching and obtained the existence of the stationary distribution on the stochastic mutualistic system. In this paper we study the ergodic stationary distribution of a stochastic Lotka–Volterra predator–prey model with regime switching using the theory and methods of [18].

The paper is organized as follows. We recall the fundamental theory and give two theorems concerning the existence of a global positive solution and moment estimation in Section 2. Section 3 investigates the persistence in mean and the extinction of the population model (1.2) and we find the threshold between them. We show the existence of a stationary distribution in Section 4. In Section 5, we present an example and figures to illustrate the main results. Finally we present some conclusions and future directions in Section 6.

2. Preliminaries

In this paper, we suppose that there is a complete probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t \geq 0}, \mathbb{P})$ with a filtration $\{\mathscr{F}_t\}_{t \geq 0}$ satisfying the usual conditions (i.e. it is right continuous and $\mathscr{F}_0$ contains all P-null sets). Let R_+^2 denote the positive zone of R^2, namely $R_+^2 = \{(x, y) \in R^2 : x > 0, y > 0\}$. For any vector $\phi = (\phi(1), \ldots, \phi(m))^T$, $\lim_{t \to \infty} \frac{1}{t} \int_0^t \phi(r(s))ds = \sum_{k \in \mathbb{S}} \pi_k \phi(k)$. Let $\hat{\phi} = \min_{k \in \mathbb{S}}\{\phi(k)\}$ and $\check{\phi} = \max_{k \in \mathbb{S}}\{\phi(k)\}$.

Lemma 2.1 ([12]). *Suppose that $z(t) \in C[\Omega \times [0, +\infty), R_+]$. If there are positive constants λ_0, T and $\lambda \geq 0$ such that*

$$\log z(t) \geq \lambda t - \lambda_0 \int_0^t z(s)ds + \sum_{i=1}^n \beta_i B_i(t)$$

for $t \geq T$, where β_i is a constant, $1 \leq i \leq n$, then $\liminf_{t \to \infty} \frac{1}{t} \int_0^t z(s)ds \geq \frac{\lambda}{\lambda_0}$, a.s.