Commun Nonlinear Sci Numer Simulat 29 (2015) 233–239

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Early warnings indicators of financial crises via auto regressive moving average models

Davide Faranda [a,*], Flavio Maria Emanuele Pons [b], Eugenio Giachino [c], Sandro Vaienti [d], Bérengère Dubrulle [e]

[a] Laboratoire des Sciences du Climat et de l'Environnement, CEA Saclay, 91191 Gif-sur-Yvette, France
[b] Department of Statistics, University of Bologna, Via delle Belle Arti 41, 40126 Bologna, Italy
[c] Studio Emanuel—Via C. Colombo 1, Torino, Italy
[d] Aix Marseille Université, CNRS, CPT, UMR 7332, 13288 Marseille, France and Université de Toulon, CNRS, CPT, UMR 7332, 83957 La Garde, France
[e] Laboratoire SPHYNX, Service de Physique de l'Etat Condensé, DSM, CEA Saclay, CNRS URA 2464, 91191 Gif-sur-Yvette, France

ARTICLE INFO

Article history:
Received 11 February 2015
Revised 16 April 2015
Accepted 5 May 2015
Available online 15 May 2015

Keywords:
Crises detection
ARMA modeling
Early warnings indicators

ABSTRACT

We address the problem of defining early warning indicators of financial crises. To this purpose, we fit the relevant time series through a class of linear models, known as auto-regressive moving-average (ARMA(p, q)) models. By running such a fit on intervals of the time series that can be considered stationary, we first determine the *typical* ARMA($\overline{p}, \overline{q}$). Such a model exists over windows of about 60 days and turns out to be an AR(1). For each of them, we estimate the relative parameters, i.e. ϕ_i and θ_i on the same running windows. Then, we define a distance Υ from such typical model in the space of the likelihood functions and compute it on short intervals of stocks indexes. Such a distance is expected to increase when the stock market deviates from its *normal* state for the modifications of the volatility which happen commonly before a crisis. We observe that Υ computed for the Dow Jones, Standard and Poor's and EURO STOXX 50 indexes provides an effective early warning indicator which allows for detection of the crisis events that showed precursors.

1. Introduction

In the late 1970s, a succession of currency crises generated interest in early warning indicators [1,2]. Over the year, the indicators spread more generally to financial and economic crisis, generating methodological debates [3,4]. Traditional statistical approaches to this issue are based on specific properties of ideal statistical systems near critical transition: critical slowing down, modifications of the auto-correlation function or of the fluctuations [5], increase of variance and skewness [6], diverging susceptibility [7–9], diverging correlation length (see the book [10] and the paper [11] for a comprehensive review). However, in many cases, these approaches fail to detect the financial crisis. First of all, such methods are *attractor* based, i.e. they assume that the system can be well described by relating the observation at the time t with the observation at the time $t + \tau$ by an empirical deterministic law describing a stationary state of the system. This approach fails in describing financial data because such processes involve a family of time scales rather than a single scale τ, [12–14]. A second origin for the failure of traditional early warning indicators is due to the presence of human feed-backs on the system i.e. the constant attempt to keep economy in a

* Corresponding author. Tel.: +33681038467.
E-mail address: davide.faranda@cea.fr (D. Faranda).

http://dx.doi.org/10.1016/j.cnsns.2015.05.002
1007-5704/

state fit to make profits. Such feed-backs create some delays between the first early warning signals and the time at which the crisis is observed. In addition, traditional early warning indicators may be inapplicable in datasets containing a small number of observations (see *e.g.* [15,16]), which is usually the case in financial time series. This suggests that indicators based on single statistical properties are not well suited for financial analysis and that crisis detection must involve indicators based on global properties of the whole stochastic process. Here, we build a class of indicators based on the auto-regressive moving-average processes of order p, q ARMA(p, q), widely used to model and forecast the behavior of financial time series. We remark that the goal of this paper will not be to find the best model to describe stock indexes and make predictions: this would require at least the estimation of fractionally integrated (ARFIMA(p, d, q)) or conditionally heteroskedastic (GARCH(p)) models, among others. We will rather assess a *typical* ARMA(p, q) model able to capture the general features of the analyzed stock market and define the early warning indicators as deviations from such a model in a suitable likelihood space. In the first part of the paper, we recall some basics on ARMA(p, q) modeling and define corresponding early-warning indicators. We then check that these indicators are able to detect the transition in theoretical financial models. We conclude the paper by presenting and discussing the results of the analysis for some stock indexes.

2. The method

Let us consider a series X_t of an observable with unknown underlying dynamics. We further assume that for a time scale τ of interest, the time series $X_{t_1}, X_{t_2}, \ldots, X_{t_\tau}$ represents a stationary phenomenon. Since X_t is stationary, we may then model it by an ARMA(p, q) process such that for all t:

$$X_t = \sum_{i=1}^{p} \phi_i X_{t-i} + \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} \tag{1}$$

with $\varepsilon_t \sim \mathrm{WN}(0, \sigma^2)$—where WN stands for white noise—and the polynomials $\phi(z) = 1 - \phi_1 z_{t-1} - \cdots - \phi_p z_{t-p}$ and $\theta(z) = 1 - \theta_1 z_{t-1} - \cdots - \theta_q z_{t-q}$, with $z \in \mathbb{C}$, have no common factors. Notice that, hereinafter, the noise term ε_t will be assumed to be a white noise, which is a very general condition [17]. For a general stationary financial time series, this model is not unique. However there are several standard procedures for selecting the model which fits at best the data. The one we exploit in this paper is the Box–Jenkis procedure [18]. We choose the lowest p and q such that the residuals of ARMA(p, q) fit are uncorrelated: to this purpose, we perform a Ljung–Box test for the absence of serial correlation (see, for example, [17]). This fixes p and q, and thus our statistical model. There are other model selection procedures based on information criteria (Bayesan or Akaike information criteria). We tested that they all give clear indications for discriminating the model and that they provide qualitatively the same results of the Box–Jenkis procedure. Intuitively, p and q are related to memory lag of the process, while the coefficients ϕ_i and θ_i represent the persistence: the higher their sum (in absolute value), the slower the system is forgetting its past history.

Our definition of early warning indicators requires first the identification of the basic ARMA($\overline{p}$,$\overline{q}$) process (with $\overline{p}$ and $\overline{q}$ fixed) which is best suited to describe a Stock index, for a time interval which can be considered stationary. This basic process plays the role of an *attractor* in the sense that it contains the information related to the dynamical properties of the system. With respect to the common attractors used in dynamical systems theory, dynamical indicators as the Lyapunov exponents are replaced by the coefficients ϕ_i and θ_j and the analogous of the attractor dimension is the number of terms p and q to be considered. We will comment on these analogies and on the possibility of choosing a reliable ARMA($\overline{p}$,$\overline{q}$) for stock market indexes in the next section. Now we turn to the ARMA based definition of the early warning indicators.

ARMA based early warning indicators. We consider a given Stock index, that is assumed to faithfully reflect the financial or economic conjuncture. In the absence of crisis, such index can be considered as stationary over a given time interval τ and can be fitted by a reference ARMA model. When a crisis approaches, the volatility of the index increases, and the best ARMA model describing the market will deviate from the basic one. The strongest the crisis, the larger the deviations will be. This suggests to introduce an early warning indicator as a suitable distance in the ARMA space from the reference model. For this, we introduce the Bayesian information criterion (BIC), measuring the relative quality of a statistical model, as:

$$\mathrm{BIC} = -2\ln \hat{L}(n, \hat{\sigma}^2, p, q) + k[\ln(n) + \ln(2\pi)], \tag{2}$$

where $\hat{L}(n, \hat{\sigma}^2, p, q)$ is the likelihood function for the investigated model and in our case $k = p + q$ and n the length of the sample. The variance $\hat{\sigma}^2$ is computed from the sample and is a series-specific quantity.

We can define a normalized distance between the reference ARMA($\overline{p}$, $\overline{q}$) and any other ARMA(p, q) model as the normalized difference between the BIC(n, $\hat{\sigma}^2$, $p+1$, q) and the ARMA($\overline{p}$, $\overline{q}$) BIC(n, $\hat{\sigma}^2$, $\overline{p}$, $\overline{q}$):

$$\Upsilon = 1 - \exp\{|\mathrm{BIC}(p+1, q) - \mathrm{BIC}(\overline{p}, \overline{q})|\}/n. \tag{3}$$

with $0 \le \Upsilon \le 1$: it goes to zero if the dataset is well described by an ARMA($\overline{p}$, $\overline{q}$) model and tends to one in the opposite case.

We have already checked that such indicators perform well in different physical systems, providing more information than the usual ones, based on the critical slow down due to the increase of correlations in the systems at the transition. These analyses have been recently published in [19] where indicators similar to Υ have been used to model different physical systems: Ising and Langevin models, climate and turbulence.