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Vibration analysis of a beam with moving support subjected to a moving mass travelling with constant and variable speed

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ABSTRACT

This study concerns with nonlinear coupled vibration analysis of a beam with moving supports under the action of a moving mass. Two different cases were studied. In the first case, the moving mass passes the beam with a constant speed, i.e. the moving mass speed is not affected by the vertical or horizontal motion of the beam. This means that the problem has two unknown variables namely the longitudinal and transverse displacements of the beam, respectively. For the second case, the relative speed of the moving mass was considered as a variable in time. In other words, in addition to the longitudinal and transverse displacements of the beam, there will be another variable, i.e. relative position of the moving mass which needs to be determined. For both cases, the coupled nonlinear governing equations of motion were derived and solved numerically for different parameters. The effect of the motion of each support, the amplitude and frequency of support excitations were also investigated.

1. Introduction

Continuous beams are common structures in modeling the behavior of bridges in highways and railways. The dynamic response of a beam under the action of a moving mass is of a great importance among engineers due to the stresses induced in the structure and the force applied to the moving mass. The influence of different types of moving loads on various structures was analyzed by Fryba [1]. The nonlinear behavior of a one-dimensional model of the disc brake pad was the subject of [2]. A strip element of the pad with arbitrary width by neglecting the disk curvature was modeled as a beam under a distributed normal load which is the force that the disc imparts to the pad. The normal force between the disc brake pad lining and rotor is a function of the relative displacement between the two bodies and is modeled by a second order polynomial. Also the frictional force between these two bodies is represented by a time-dependent load. The governing equation of the transverse motion was discritized by the Galerkin method. Frequency response curves were obtained using a spectral balance method and considering two modes in the solution procedure. Chaotic vibration was predicted due to the existence of nonlinear contact force between the disc brake pad lining and rotor. The chaotic behavior was investigated by utilizing the Poincare´ maps, Fourier spectra, and Lyapunov exponents. The nonlinear dynamic response of an inclined beam subjected to a concentrated vertical force traveling with a constant velocity was investigated in [3]. Two nonlinear coupled equations of longitudinal and transverse motion of the beam were solved using the multiple scales method. The results for various load velocity ratios were compared to those obtained from traditional linear solution. The effect of various parameters such as linear viscous damping, the velocity of the moving load, beam inclination angle was also studied. Also the planar motion path of the projectile after leaving the inclined beam was investigated and it was compared to the solution in which the beam is assumed rigid.

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Depending on the velocity and acceleration of the moving part, the model of moving load should be substituted by the moving mass one. In this case, the effect of inertial terms of the moving part such as Coriolis, centripetal and relative accelerations is considered. Investigation of several parameters such as mass sprung effect, viscoelastic effect, different elastic structures and boundary conditions are carried out in [4–7]. Vibration of an elastic bridge loaded by a moving train modeled as a moving elastic beam was studied in [8]. The connection of two bodies was assumed to be rigid and the effect of the mass and stiffness of the train was investigated. The bending moment and shear force in the bridge were obtained using symbolic computation. A technique to obtain numerical solution for nonlinear dynamic systems undergoing large deflection was developed in [9]. The system considered here is the nonlinear interaction between a beam and a moving spring-mass. First, the equations of motion for the large amplitude of motion of the beam were derived. Then these equations were solved by applying a new method in which average acceleration approach is utilized to reduce nonlinear ODEs to nonlinear algebraic equations. The obtained algebraic equations were solved by an iterative scheme. A small-scale bridge model subjected to a moving concentrated mass was investigated in [10]. A simply supported Euler–Bernoulli beam was used and the deflection of the beam was expanded in a series of eigenfunctions. An experimental model in small scale was tested and the results agreed well with theoretical predictions.

Among numbers of studies, there are few ones in which the effect of the variable velocity has been taken into account. In this case, the relative position of the moving part is not a constant speed process and may be other processes like the constant acceleration one. Linear dynamic response of a simply supported elastic single span beam subjected to a moving constant load with variable speed was also studied in [11]. It was mentioned how acceleration and deceleration caused by an external force can affect the behavior of the beam under the action of a single load or a real vehicle model. In this study, the acceleration and deceleration processes are considered as a constant acceleration process. Dynamic response of pre-stressed Rayleigh beam resting on elastic foundation and subjected to masses traveling at varying velocity was presented in [12]. Analytical solution for the deflection of the beam subjected to both a moving mass and a moving load with constant acceleration was obtained. The effect of parameters such as axial load and rotary inertia was studied in both of the cases. The study considered a constant acceleration process and therefore the speed of the moving mass was kept constant. An analytical and numerical study of the dynamic response of an initially curved beam subjected to multiple moving masses was the objective of [13].The equations of motion of the system containing rigid masses rolling on an initially curved Euler–Bernoulli beam were obtained using Newtonian approach. It was shown that the initial curvature of the beam can have a great influence on the dynamic response even if the initial imperfection of the beam is small. Also it was concluded that the initial velocity of the mass, the applied force on the mass, and the friction between the mass and the beam affect the amplitude of the response of the system. The effect of an eccentric path along a finite inextensible beam on the dynamic response of the system was investigated in [14]. It was indicated that for the concave upward eccentric path, the amplitude of eccentricity amplifies both the amplitude of the trajectory of mass and the displacement of the beam. However for the convex upward eccentric path, the amplitude of eccentricity attenuated the displacement of the beam. Vibration of elastic homogeneous isotropic beam with general boundary conditions traversed by moving loads was studied in [15]. Solutions for the response of beams under the action of a moving force travelling with constant speed and constant acceleration in accelerating and decelerating motions were presented in close-form. The effects of different parameters such as boundary conditions and damping were also studied.

The vibration of suspended bridges under the action of multiple moving loads and vertical support motions was the subject of [16,17]. A row of equidistant moving forces was utilized in order to model the traffic load on a suspended bridge subjected to vertical motions of supports due to the consideration of earthquake. The effect of different arrival time was also studied and the results show amplification in the response of the long-span bridge and increase in the velocity of the moving train.

The three-dimensional dynamic behavior of a beam on a Winkler-type elastic foundation subjected to a moving vehicle and random lateral excitations was investigated in [18]. The lateral excitation was modeled in two cases namely, a lateral moving load with random intensity and a moving mass just in lateral direction of the beam in order to simulate the wind excitation and earthquake excitation, respectively. The effect of support stiffness and velocities of the vehicle on the response of the beam were studied. A linear optimal control algorithm was proposed and its efficiency in the suppression of the response was verified. The response spectrum of an Euler–Bernoulli beam subjected to a moving mass and horizontal support excitation was explored in [19]. The effect of the inertial terms of the moving mass on the natural frequencies of the beam was studied in order to avoid dynamic instability.

The important contribution of this study lies in investigating the effect of base movement on the coupled nonlinear vibration of a beam under the action of a moving mass. Nonlinearity arises from large deformation assumption in beam theory. It was shown that such motion has a great influence on the dynamic behavior of the beam. Two different cases, one with a moving mass travelling at constant speed and the other one with a mass moving with variable speed are studied. First, the equations of motion for both cases are derived and then the nonlinear coupled governing equations are solved numerically. The effect of amplitudes and frequencies of the harmonically moving supports are studied. For the second case, the relative position of the moving mass on the beam is considered as a variable and the effect of base motion on this variable is also investigated.

2. Theory

Consider a simply supported Euler–Bernoulli beam of span l, mass per unit length m and the bending stiffness of EI, where E is Young's modulus and I is the second moment of inertia for the cross sectional area, as it is depicted in Fig. 1. The beam is traversed by a moving mass M which is at a position $s(t)$ and its supports are moving horizontally due to the movement of the base shown by $u_1(t)$ and $u_2(t)$, respectively.