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Analytical study on the motions around equilibrium points of restricted four-body problem

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ABSTRACT

We investigate the motions around the equilibrium points of restricted four-body problem, where the three primaries with unequal masses constitute a Lagrangian configuration which is linearly stable. About the dynamical model studied, there are eight non-collinear equilibrium points, three of them are stable and the remaining ones are unstable. The linear dynamics of these equilibrium points state that there are center and hyperbolic manifolds in the vicinity of unstable equilibrium points, and there are long, short and vertical periodic orbits around stable equilibrium points. Based on the nonlinear equations of motion, the general solutions around equilibrium points are expanded as formal series of several amplitude parameters. Lissajous orbits around unstable equilibrium points are expressed as formal series of the in-plane and out-of-plane amplitudes. Invariant manifolds around unstable equilibrium points are expanded as formal series of four amplitudes, two of them correspond to hyperbolic dynamics and the remaining ones correspond to center dynamics. The motions around stable equilibrium points are expressed as formal series of long, short and vertical periodic amplitudes. By means of Lindstedt–Poincaré method, series solutions are constructed up to a certain order. The advantage of the series solutions constructed lies in that the motions around equilibrium points can all be parameterized. At last, the practical convergence has been computed in order to check the validity of the series expansions constructed.

1. Introduction

A rough estimate states that approximately two-thirds of the stars in our Galaxy belong to the multistellar systems, and about one-fifth of these multistellar systems are triple systems [1]. Up to date, hundreds of exoplanetary systems have been detected, among these systems more than 400 systems are multi-planet systems [2]. In our Solar system, the Sun–planet–satellite and planet–satellite–satellite systems are all multi-body systems. To study the motion of an infinitesimal particle in the multi-body system, restricted N-body model is usually adopted, such as circular restricted three-body problem (CRTBP) and restricted four-body problem (RFBP). For the dynamical model RTBP, a lot of researchers have contributed to the investigation on this problem, and abundant literature can be found. For the dynamical model RFBP, the three primaries constitute a central configuration and the motion of the fourth massless body is governed in the gravitational field generated by the three primaries. According to the type of central configuration (given by the relative equilibrium solution), RFBP has two versions [3]: triangular configuration (Lagrangian configuration) and collinear configuration (Eulerian configuration). For the triangular configuration, the primaries lie always at the apices of an equilateral triangle. For the collinear configuration, the primaries always lie on a straight line, where the massive body is located at the origin and the remaining two with equal masses lie at two sides of the massive one. In order

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to explain some complex phenomenon presented in our Solar system, exoplanetary system and multistellar system, it is of great significance to study the dynamics in the dynamical model RFBP, such as the stability, the region of motion, dynamics around equilibrium points, periodic and asymptotic trajectories.

About the model RFBP corresponding to the collinear configuration, there are six equilibrium points, four of them lie on the x-axis and the remaining two lie on the y-axis. Papadakis [1] computed the invariant stable and unstable manifolds around the collinear libration points numerically, and calculated the symmetric and non-symmetric homo- and hetero-clinic asymptotic orbits in RFBP. Similar to CRTBP, there are families of periodic orbits in RFBP, which could assist us to understand the structure of solution. Kalvouridis et al. [4] dealt with the photo-gravitational version of RFBP, and investigated the effect of radiation on the distribution of periodic orbits, the stability and evolution of the families of periodic orbits. For the same system discussed in [4], Kalvouridis and Hadjifotinou [5] have studied the bifurcations of three-dimensional periodic orbits from two-dimensional orbits by taking advantage of the method of vertical critical stability. The phenomenon "blue sky catastrophe" shown in circular restricted three-body problem (CRTBP) has been extended to RFBP by Burgos-García and Delgado [6].

For the RFBP corresponding to triangular configuration, the existence and number of equilibrium points depend on the distribution of the mass parameters. Baltagiannis and Papadakis [7] studied the dependence of equilibrium points and their stability on the mass parameters of the primaries, and concluded that when the three primaries have equal masses, there are ten unstable equilibrium points, four are collinear and the remaining ones are non-collinear; when two of the primaries have equal masses, the dynamical model admits two or four collinear and four or six non-collinear equilibrium points; when the primaries have unequal masses, there are eight or ten non-collinear equilibrium points. In [8], Baltagiannis and Papadakis studied the evolution of the families of periodic orbits in RFBP and the stability of periodic orbits in three cases: all primaries with equal masses, two primaries with equal masses, and all primaries with unequal masses. For the Sun–Jupiter–Trojan asteroid–spacecraft system, the families of periodic orbit around the Asteroid and/or the Jupiter were calculated by Baltagiannis and Papadakis [9]. The periodic orbits in the restricted four-body problem with two equal masses were explored in [10]. In [11], a low-thrust version of RFBP has been formulated to investigate the potential applications in the Sun–Jupiter–Trojan asteroid–spacecraft system. For the restricted four-body problem with oblateness effects and photo gravitation, the stability region of equilibrium points have been investigated by Kumari and Kushvah [12], and Papadouris and Papadakis [13].

In our Solar system, the version of RFBP corresponding to triangular configuration can be applied to Saturn–Tethys–Telesto–spacecraft, Saturn–Tethys–Calypso–spacecraft and Saturn–Dione–Helen–spacecraft systems [14], besides the Sun–Jupiter–Trojan asteroid–spacecraft system discussed in [9,11]. Dynamical model RFBP is an autonomous system, and the dynamics around equilibrium points of RFBP are similar to the ones of CRTBP. Therefore, the analytical techniques developed for studying the dynamics around equilibrium points of CRTBP can also be applied to investigate the dynamics around the equilibrium points of RFBP. For CRTBP, Richardson [15] derived the third order analytical solutions of halo orbits around collinear libration points, which provide an estimation of the initial state to numerically generate the accurate halo orbit. Jorba and Masdemont [16] expanded the Lissajous and halo orbits around collinear libration points of CRTBP as formal series of two amplitudes, and semi-analytically constructed high-order solutions with the assistance of computer. Due to the hyperbolic dynamics around collinear libration points in CRTBP, there are stable and unstable manifolds around collinear libration points, which play an important role in the design of transfer trajectory [17–19]. Masdemont [20] expanded the invariant manifolds associated with Lissajous and halo orbits as formal series in powers of four amplitude parameters, two of them are the amplitudes corresponding to hyperbolic manifolds, and the remaining two are amplitudes corresponding to center manifolds. By taking advantage of the series expansions constructed in [20], the general motions around collinear libration points of CRTBP can be parameterized by several related parameters. Triangular libration points of CRTBP are stable when the mass parameter is less than Routh's critical value [21], and in the vicinity of triangular points, there are three types of periodic orbits: long, short and vertical periodic orbits. Lei and Xu [22] semi-analytically constructed high-order series solutions around the triangular libration points in CRTBP by means of Lindstedt–Poincaré method. When the eccentricity of the primaries is different from zero, such a system is called elliptic restricted three-body system (ERTBP), which could approximate the three-body systems in our Solar system more accurately. Hou and Liu [23] constructed high-order series solutions of Lissajous and halo orbits around collinear libration points in ERTBP, and Lei et al. [24] and Lei and Xu [25] expanded the invariant manifolds around collinear libration points and the motions around triangular libration points of ERTBP as formal series of several related amplitude parameters.

For the dynamical model RFBP where the three primaries form a stable Lagrangian configuration, there are eight non-collinear equilibrium points, three of them are stable, and the remaining ones are unstable. In this paper, we will analytically investigate the motions around unstable and stable equilibrium points by the similar methods mentioned above for studying the dynamics around equilibrium points of RTBP [16,20, 22–25].

The remainder of this paper is structured as follows. Section 2 introduces the dynamical model RFBP and presents the basic dynamics around equilibrium points. Sections 3 and 4 discuss how to construct high-order series expansions of Lissajous orbits and invariant manifolds around unstable equilibrium points, respectively. Section 5 presents the series expansions of periodic or quasi-periodic motions around stable equilibrium points. In Section 6, the simulation results are summarized. At last, the conclusions are drawn in Section 7.

2. Basic dynamics of restricted four-body problem

The motions of a massless particle (such as a spacecraft or an asteroid) are investigated under the gravitational field generated by the three primaries with masses m_1, m_2 and m_3, forming an equilateral triangle configuration, but the gravitational influence