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The role of variability in transport for large-scale flow dynamics

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We develop a framework to study the role of variability in transport across a kinematically-defined boundary defined as a streamline in a reference flow. Two complementary schemes are presented: a graphical approach appropriate for analyzing specific cases of variability, and an analytical approach for analyzing the effect of general fluid properties on variability. Spatio-temporal nonlinear interaction between dynamic variability and the reference flow leads to flux variability that governs the transport processes. A characteristic length-scale of dynamic and flux variability can be expressed with the units of time using the flight time of the trajectory along the kinematically defined boundary. The characteristic time-scale of the flux variability is that of dynamic variability with the units of time. The non-dimensional ratio of the two characteristic scales is shown to be a critical parameter for evaluating the effectiveness of variability on transport. The pseudo-lobe sequence along the reference streamline describes spatial coherency of transport. The emergence of the pseudo-lobe sequence is likely to be synchronous with the flux variability. Once a pseudo-lobe sequence is formed, the characteristic length-scale of the flux variability regulates the width of the pseudo-lobes. In contrast, for transport over a fixed time interval and spatial segment, the characteristic time-scale of the dynamic variability regulates the width of the pseudo-lobes. Using a kinematic model, we demonstrate the framework for two types of transports in a blocked flow of the mid-latitude atmosphere: across the meandering jet axis and between the jet and recirculation cell.

1. Introduction

1.1. Geophysical flows and dynamic variability

Large-scale geophysical flows are approximately two-dimensional and nearly incompressible. Quite often their time evolution may be described as unsteady fluctuations around a prominent time-mean field [8,17,19]. The instantaneous velocity field $\mathbf{u} = (u, v)^T$ of such flows at time t in two-dimensional $\mathbf{x} = (x, y)^T$ space can be written as:

$$\mathbf{u}(\mathbf{x}, t) = \bar{\mathbf{u}}(\mathbf{x}) + \mathbf{u}'(\mathbf{x}, t), \tag{1}$$

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where $\bar{\{\cdot\}}$ and $\{\cdot\}'$ stand for time-averaged ("reference") and residual fluctuation ("anomaly") fields, respectively. The anomaly field $\mathbf{u}'(\mathbf{x}, t)$ of a large-scale geophysical flow may contain energetic dynamic signals that are spatially and temporally coherent. A commonly used technique for the detection of such coherent evolution is the empirical orthogonal function (EOF), or principal component (PC) analysis. By the application of the singular value decomposition to the anomaly field, the EOF leads to an orthogonal set of spatio-temporal modes:

$$\mathbf{u}'(\mathbf{x},t) = \sum_{i=1}^{i_{\max}} \tilde{\mathbf{u}}_i(\mathbf{x},t) = \sum_{i=1}^{i_{\max}} \sigma_i \mathbf{u}_i(\mathbf{x}) f_i(t), \tag{2}$$

where $i_{\max}$ is the dimension of the anomaly field and $\tilde{\{\cdot\}}$ stands for the spatial–temporal decomposition throughout this paper. Both the spatial PC $\mathbf{u}_i(\mathbf{x})$ and the temporal PC $f_i(t)$ are normalized so that the (ordered, positive) variance $(\sigma_i)^2$ with $(\sigma_i)^2 \geq (\sigma_{i+1})^2 \geq 0$ reflects the statistical significance of the mode i. The relative variance of the dynamic variability explained by the mode i is $(\sigma_i)^2 / \sum_i^{i_{\max}} (\sigma_i)^2$. Quite often the first few modes dominate and explain most of the variance of the dynamic variability. We note that another spatio-temporal decomposition technique uses spectral analysis, such as a normal mode analysis decomposition [10].

As a single mode, $\tilde{\mathbf{u}}_i(\mathbf{x},t) = \sigma_i \mathbf{u}_i(\mathbf{x}) f_i(t)$ describes a standing features of $\mathbf{u}_i(\mathbf{x})$ in space that pulsates with $f_i(t)$ in time. We call the spatial features in $\mathbf{u}_i(\mathbf{x})$ the "dynamic eddies of mode i". A positive or a negative eddy respectively corresponds to a local region where flow is anti-cyclonic (clockwise) or cyclonic (counter-clockwise). Characteristic length-scale L_i of the mode i corresponds to the typical size of the dynamic eddies. Temporal coherency is defined by the positive and negative phases of $f_i(t)$ based on its sign. If the variability of the mode is nearly periodic, then:

$$f_i(t + T_i) \approx -f_i(t). \tag{3}$$

Thus starting with t_0 when $f_i(t_0)$ is local maximum, four phases of the standing features in $\tilde{\mathbf{u}}_i(\mathbf{x},t)$ over one cycle $2T_i$ are $\tilde{\mathbf{u}}_i(\mathbf{x},t_0)$, $\mathbf{0}$, $-\tilde{\mathbf{u}}_i(\mathbf{x},t_0)$, and $\mathbf{0}$ because $f_i(t_0) \approx -f_i(t_0 + T_i)$ and $f_i(t_0 + T_i/2) \approx f_i(t_0 + 3T_i/2) \approx 0$. Typically the lower the mode i is, and hence the larger the variance $(\sigma_i)^2$ is, the larger the characteristic scales for both $\mathbf{u}_i(\mathbf{x})$ and $f_i(t)$, i.e., $L_i \geq L_{i+1}$ and $T_i \geq T_{i+1}$ for $(\sigma_i)^2 \geq (\sigma_{i+1})^2$.

The anomaly field $\mathbf{u}'(\mathbf{x}, t)$ may also exhibit propagation of energetic dynamic eddies, which may be described by a pair of two dynamic modes that have the same characteristic length-and time-scales as these dynamic eddies. For an illustration, consider a simple propagating wave in the channel:

$$\mathbf{u}'(\mathbf{x},t) = \epsilon\pi \begin{pmatrix} l & \sin k\pi(x - bt) & \cos l\pi y \\ k & \cos k\pi(x - bt) & \sin l\pi y \end{pmatrix}. \tag{4a}$$

This wave has the characteristic scales $L = 1/b$ and $T = 1/kb$, with the phase speed b in x. The application of the EOF yields analytically to a pair of the two modes:

$$\tilde{\mathbf{u}}_1(\mathbf{x},t) = \sigma_1 \mathbf{u}_1(\mathbf{x}) f_1(t) = \epsilon\pi \begin{pmatrix} l & \sin k\pi x & \cos l\pi y \\ -k & \cos k\pi x & \sin l\pi y \end{pmatrix} \cos kb\pi t \tag{4b}$$

$$\tilde{\mathbf{u}}_2(\mathbf{x},t) = \sigma_2 \mathbf{u}_2(\mathbf{x}) f_2(t) = \epsilon\pi \begin{pmatrix} l & \cos k\pi x & \cos l\pi y \\ k & \sin k\pi x & \sin l\pi y \end{pmatrix} \sin kb\pi t, \tag{4c}$$

where $\mathbf{u}'(\mathbf{x},t) = \tilde{\mathbf{u}}_1(\mathbf{x},t) + \tilde{\mathbf{u}}_2(\mathbf{x},t)$. The two modes have the same scales as the original, $L_1 = L_2 = L$ and $T_1 = T_2 = T$. Both spatial and temporal PCs are phase shifted by a half, due to the orthogonality of the PCs.

Conversely, the sum of two consecutive modes, $\tilde{\mathbf{u}}_i(\mathbf{x},t) + \tilde{\mathbf{u}}_{i+1}(\mathbf{x},t)$, can represent propagation of dynamic eddies if the variance and scales of the two modes match, i.e., $(\sigma_i)^2 \approx (\sigma_{i+1})^2$, $L_i \approx L_{i+1}$, and $T_i \approx T_{i+1}$. It naturally follows by the orthogonality of the PCs that the dynamic eddies in $\mathbf{u}_i(\mathbf{x})$ and $\mathbf{u}_{i+1}(\mathbf{x})$ are spatially phase-shifted by $L_i/2 (\approx L/_{i+1}/2)$, while $f_i(t)$ and $f_{i+1}(t)$ are temporally phase-shifted by $T_i/2 (\approx T_{i+1}/2)$. For t convenience later in Section 4, we chose:

$$f_i(t) \approx f_{i+1}(t + T_i/2). \tag{5}$$

If the EOF returns $f_i(t) \approx f_{i+1}(t - T_i/2) \approx -f_{i+1}(t + T_i/2)$, then the transformation $\mathbf{u}_{i+1}(\mathbf{x}) \to -\mathbf{u}_{i+1}(\mathbf{x})$ and $f_{i+1}(\mathbf{x}) \to -f_{i+1}(\mathbf{x})$ would satisfy the desired relation (5) given the $f_i(t)$ relation in (5). Then $\tilde{\mathbf{u}}_i(\mathbf{x},t) + \tilde{\mathbf{u}}_{i+1}(\mathbf{x},t)$ exhibits propagation of dynamic eddies over $2L$ in $2T$ during one cycle, where $L \approx L_i \approx L_{i+1}$ and $T \approx T_i \approx T_{i+1}$. Starting at t_0 when $f_i(t_0)$ is local maximum and hence $f_{i+1}(t_0) \approx 0$, four phases of propagating dynamic eddies in $\tilde{\mathbf{u}}_i(\mathbf{x},t) + \tilde{\mathbf{u}}_{i+1}(\mathbf{x},t)$ during one cycle $2T$ are $\tilde{\mathbf{u}}_i(\mathbf{x},t_0)$, $\tilde{\mathbf{u}}_{i+1}(\mathbf{x},t_0 + T/2)$, $-\tilde{\mathbf{u}}_i(\mathbf{x},t_0)$, and $-\tilde{\mathbf{u}}_{i+1}(\mathbf{x},t_0 + T/2)$ because $f_i(t_0) \approx f_{i+1}(t_0 + T/2) \approx -f_i(t_0 + T) \approx -f_{i+1}(t_0 + 3T/2)$ and $f_{i+1}(t_1) \approx f_i(t_0 + T/2) \approx f_{i+1}(t_1 + T) \approx f_i(t_0 + 3T/2) \approx 0$. In the Rossby traveling wave example, the two modes defined by (4b) and (4c) together represent the propagating wave as in (4a). In this study, we denote such a pair $[i, i+1]$ by the subscripts in brackets, i.e., $\tilde{\mathbf{u}}_{[i,i+1]}(\mathbf{x},t) = \tilde{\mathbf{u}}_i(\mathbf{x},t) + \tilde{\mathbf{u}}_{i+1}(\mathbf{x},t)$.

1.2. Transport

Transport plays important roles in a wide range of geophysical systems. For a large-scale geophysical flow in which $\bar{\mathbf{u}}(\mathbf{x})$ dominates $\mathbf{u}(\mathbf{x}, t)$, transport occurs mainly streamwise with respect to $\bar{\mathbf{u}}(\mathbf{x})$. We call a streamline defined by $\bar{\mathbf{u}}(\mathbf{x})$ a "reference