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Wavelet modeling and prediction of the stability of states: the Roman Empire and the European Union

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ABSTRACT

How can the stability of a state be quantitatively determined and its future stability predicted? The rise and collapse of empires and states is very complex, and it is exceedingly difficult to understand and predict it. Existing theories are usually formulated as verbal models and, consequently, do not yield sharply defined, quantitative prediction that can be unambiguously validated with data. Here we describe a model that determines whether the state is in a stable or chaotic condition and predicts its future condition. The central model, which we test, is that growth and collapse of states is reflected by the changes of their territories, populations and budgets. The model was simulated within the historical societies of the Roman Empire (400 BC to 400 AD) and the European Union (1957–2007) by using wavelets and analysis of the sign change of the spectrum of Lyapunov exponents. The model matches well with the historical events. During wars and crises, the state becomes unstable; this is reflected in the wavelet analysis by a significant increase in the frequency $\omega(t)$ and wavelet coefficients $W(\omega, t)$ and the sign of the largest Lyapunov exponent becomes positive, indicating chaos. We successfully reconstructed and forecasted time series in the Roman Empire and the European Union by applying artificial neural network. The proposed model helps to quantitatively determine and forecast the stability of a state.

1. Introduction

Studying the history of a state and quantitatively determining its stability allows extrapolation of this knowledge to modern societies. It is well known that the expansion of Ancient Rome passed through all stages of civilization development: origin, heyday and decline. A well established management system made it possible to gradually expand the boundaries of the state and to rule the entire Mediterranean region [1]. Ancient Rome attracts attention because it was the foundation on which modern society is based. Several aspects of that civilization have been studied, including economic policies [2–4] and the influence of climatic changes on the population of Europe [5]. Population density and migration also affect the

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development of a state [6]. Historians and social scientists have proposed several theories to explain the expansion and collapse of Ancient Rome [7–9]. However, because existing theories are usually formulated based on verbal models, the causal mechanisms and processes underlying these theories are not always explicit. In addition, it is always a challenge to forecast stability of human societies. Therefore, creating an appropriate mathematical model will help to quantitatively determine the development of an ancient civilization as well as to forecast the stability of modern societies. In this work we created a model based on time-series of the changes in territory (Fig. 1B, D and F) and population of Ancient Rome. We also simulated development of a current society, namely the European Union (EU), based on the time-series in the change of its territory (Fig. 1A, C and E), population, and budget. This model is based on the methods of non-linear dynamics and includes wavelets of Morlet [10] and Haar [11], and Lyapunov exponents. Wavelet analysis can be used as a mathematical microscope to explore the historical process at a given time point, while Lyapunov exponents give an idea of the historical process (i.e., development of the state) as a whole and determines whether it is in a stable or chaotic condition. The proposed model accurately described the whole historical time line of existence of the Roman Empire from its origin to its decline, and of the development of the EU, and quantitatively determined their stability. In addition, artificial neuronal networks enabled us to reconstruct and forecast time-series occurring in the Roman Empire as well as in the EU.

2. Materials and methods

2.1. Wavelet analysis

Wavelet analysis is applied to time series analysis of various phenomena. In physics, for example, it is used to analyze data on the flow of sunlight, solar wind, surface structure, and galaxies [12,13]. The wavelets are versatile in application and can describe any existing signal [14]. The undeniable advantage of wavelet analysis is its ability to analyze the internal structure of the time series and its changes through localization in time and in frequency space at any desired point in time and at different scales. In this study, the historical events are presented in the form of time series. Ancient Rome provides a complete picture of the dynamics of development and decline of a state. We included analysis of the territory of the state in time during the Roman civilization from 400 BC till 400 AD, and changes in the population of the state from 400 BC till 400 AD [15]. Throughout the history of the Ancient Roman civilization, periods of prosperity and stability were followed by periods of social upheaval and war, including civil wars. The development of civilization is a dynamic system. The periods of stability of the state are defined as a stable state of the system, and the periods of war and social upheaval are defined as a chaotic state of the system.

For the EU we analyzed the following elements: the change in time of territory and population from 1957 till 2007, and an important economic component, the budget, from 1957, when the European Economic Community (EEC) and the European Atomic Energy Community (Euratom) were established, up to 2007 (http://ec.europa.eu/budget/biblio/documents/2013/2013_en.cfm). The same signal can be analyzed by different wavelets. For example, wavelet of Morlet does not respond to changes in the time series of the budget and the population of the EU, whereas the Haar wavelet allows measurement of these parameters.

We studied the historical time series by using wavelets of Morlet, Meyer, Gauss 1, Daubechies, Haar, Mexican hat and French hat [16]. However, the most meaningful analysis was achieved by using the Haar (A) and Morlet (B) wavelets:

A:

$$\Psi(\mathrm{t}) = \begin{cases} 1, & 0 \leqslant t < 0.5 \\ -1, & -0.5 \leqslant t < 1 \\ 0, & t < 0, t \geqslant 1 \end{cases}$$

B:

$$\psi(t) = \exp(i\,2\pi t)\exp\left(-\frac{t^2}{2}\right)$$

Under the time series we understand the one-dimensional signal, for example, the time dependence of the change in the territory of the state in time. The result of wavelet-transformation is wavelet-coefficients $W(t,\omega)$. The wavelet transformation is the dot product of the analyzed signal and the analyzing wavelet. Thus, wavelet coefficients contain information about both the signal and the wavelet. The parameter t is time (years), which is a shift parameter. It determines the focus point of a "mathematical microscope" (wavelet) and the scaling factor ω, the frequency. The "optical quality" of the microscope is determined by the choice of the wavelet basis, ψ. In this case, we used the wavelets of Haar and Morlet.

The wavelet spectrum of a one-dimensional signal is a surface in a three-dimensional space $W((t,\omega),t,\omega)$. Visualization of the wavelet coefficients of the spectrum allows us to see the evolution of the signal. For example, increases in the frequencies correspond to the local maxima of the time series. It also allows us to draw some conclusions about the energy component of the system. The energy of the system is the energy of the signal, which is the value

$$E_w = \int_{t_1}^{t_2} |W(t,\omega)|^2 dt \text{ on the interval } t_1 < t < t_2.$$