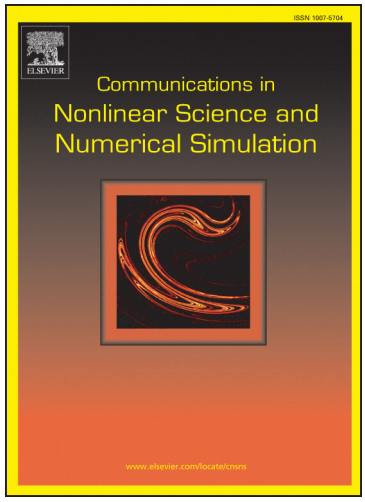

A new approach on fractional variational problems and Euler-Lagrange equations

F. Bahrami, H. Fazli, A. Jodayree Akbarfam

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A new approach on fractional variational problems and Euler-Lagrange equations

F. Bahrami, H. Fazli*, A. Jodayree Akbarfam
Faculty of Mathematical Sciences, University of Tabriz, Tabriz, Iran.

Abstract. In this paper we generalize fractional variational problems in $[a, b]$. We allow for the possibility that functions in the space of solution for the optimization problem can blow up at boundary points. The appropriate fractional derivative spaces are introduced and a compact embedding theorem demonstrated. We prove the existence of minimizers for the variational problems which satisfy the Euler-Lagrange equations with Riemann-Liouville boundary conditions. Our method is based on the fractional calculus of variations. An example is given to illustrate the results.

Keywords: Fractional calculus, Fractional variational problems, Variational solutions, Riemann-Liouville initial and boundary conditions, Fractional Euler-Lagrange equations.

2010 Mathematics Subject Classification. 34A08, 70H03.

1 Introduction

Fractional calculus is nowadays a useful field of mathematics, with many applications in physics and engineering, see, e.g., [9, 16, 18, 22, 25]. The fractional operators are non-local, therefore they are suitable for constructing models possessing memory. They provide an excellent tool for the description of complex quantum systems, dissipation, and long-range phenomena that cannot be well described using ordinary differential and integral operators, see, e.g., [11, 18, 21, 28]. Applications of fractional calculus are found in classical and quantum mechanics, field theories, variational calculus, and optimal control, see, e.g., [11, 15, 19].

Fractional calculus of variations is a recent field initiated in 1996-97 with the works of Riewe [28,29]. It is still in the initial stages and many aspects of this theory need to be explored. A fractional variational problem (FVP) is a problem in which either the objective functional or equations of constraints, or both, contain at least one fractional operator term. These problems are not only an interesting field of applied mathematics, but also identify an important class of initial and boundary value problems with

*Corresponding author.
E-mail addresses: fxb20@psu.edu (F. Bahrami), h_fazli@tabrizu.ac.ir (H. Fazli), akbarfam@yahoo.com (A. Jodayree Akbarfam).