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A stage structure pest management model with impulsive state feedback control

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ABSTRACT

A stage structure pest management model with impulsive state feedback control is investigated. We get the sufficient condition for the existence of the order-1 periodic solution by differential equation geometry theory and successor function. Further, we obtain a new judgement method for the stability of the order-1 periodic solution of the semi-continuous systems by referencing the stability analysis for limit cycles of continuous systems, which is different from the previous method of analog of Poincarè criterion. Finally, we analyze numerically the theoretical results obtained.

1. Introduction

Banana leaves diseases are divided into epiphyte and virus. Banana bunchy top disease (i.e., Prawn banana, Green banana, Banana) is one of virus diseases, caused by Banana bunchy top virus. Banana farmers call it as an incurable disease. Banana aphids are the primary propagation medium of banana virus diseases. The development of banana aphids includes eggs and adult (winged form) stages. Eggs do not carry and spread virus, but the genital organs of adult have developed fully and can oviposit, and that adult have grew wings, can transmit virus to healthy plants through crawling, migrating and airflow after piercing and sucking the virus of diseased plants. Therefore infected adult have strong infective power.

To avoid the outbreak of banana aphids, we will use ovicides to kill eggs, or use insecticides to kill adult [1–3]. Usually ovicides and insecticides have specificity, for example, eggs will not be killed if we use insecticides (such as 2000 to 2500 times dilution of acetamiprid 3% EC, 15,000 times dilution of imidacloprid 70% WG, 1000 times dilution of omethoate 40% EC, 2500 to 3000 times dilution of sumicidin 20% EC and so on) to kill adult.

In pest management, we spray pesticides only when pest density increases to the certain level called ET (Economic Threshold, i.e., pest population density at which control measures should be adopted to prevent an increasing pest population from reaching the economic injury level). ET is the index of pest density. Crop output will not decrease much when pest density is lower than ET, thus we need not adopt any control measure. Once pest density rises to ET, some measures must be carried out to prevent EIL(economic injure tolerate level) happening. In the paper, we will spray pesticides

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which can kill adult but not kill eggs. Therefore, a stage structure pest management model with impulsive state feedback control is constructed as follows:

$$\begin{cases} \left.\begin{aligned} \frac{dx}{dt} = ay - bx = P(x,y), \\ \frac{dy}{dt} = cx - dy = Q(x,y), \end{aligned}\right\} & y < y^*, \\ \Delta y = -\beta y, & y = y^*, \end{cases} \tag{1.1}$$

where $x(t), y(t)$ denote the proportions of immature and mature pest at time t, respectively, a, c denote the transformation rate from immature to mature, b, d denote the death rate of immature and mature, respectively, a, b, c, d are positive constants, $0 < \beta < 1$ is the ratio of killing mature pests by spraying pesticides, y^* denotes ET. This model describes that in productive practice for controlling mature pests, people always take such a strategy that when the mature pests arrive at a given ET y^* they will begin to kill the mature pests with chemical pesticides.

At present, for impulsive state feedback control systems, the sufficient condition for the existence and the orbitally asymptotically stability of the order-1 periodic solutions have been obtained by differential equation geometry theory, the method of successor function and analog of Poincarè Criterion [4–11]. In this paper, we try to obtain a new judgement method for the stability of the order-1 periodic solution by referencing the stability analysis of limit cycles for continuous systems.

In the next section, we give some preliminaries. In Section 3, we get the sufficient condition for the existence of the order-1 periodic solution of system (1.1) by differential equation geometry theory and successor function. In Section 4, referencing the stability analysis of the limit cycles for continuous dynamic systems, we prove the order-1 periodic solution of system (1.1) is orbitally asymptotically stable under some conditions. In Section 5, we analyze numerically the theoretical results obtained.

2. Preliminaries

Definition 2.1 [12]. Suppose impulsive state differential equation

$$\begin{cases} \frac{dx}{dt} = P(x,y), \quad \frac{dy}{dt} = Q(x,y), \quad (x,y) \notin M\{x,y\}, \\ \Delta x = \alpha(x,y), \quad \Delta y = \beta(x,y), \quad (x,y) \in M\{x,y\}, \end{cases} \tag{2.1}$$

whose solution mapping composes the system called as semi-continuous dynamic system, denoted by (Ω, f, φ, M). Set initial point of mapping $p \in \Omega = R_2^+ \setminus M\{x,y\}$, φ is a continuous mapping, $\varphi(M) = N$, φ is called as impulse mapping, where $M(x,y)$ and $N(x,y)$ are straight lines or curves on the plane $R_2^+ = \{(x,y) \in R^2 : x \geqslant 0, y \geqslant 0\}$, $M\{x,y\}$ denotes impulse set, $N\{x,y\}$ denotes phase set.

In system (1.1), impulse set $M = \{(x,y) \in R_2^+ | x \geqslant 0, y = y^*\}$, impulse mapping $\varphi : (x,y) \in M \to (x, (1-\beta)y^*) \in R_2^+$, phase set $N = \varphi(M) = \{(x,y) \in R_2^+ | x \geqslant 0, y = (1-\beta)y^*\}$. Therefore, system (1.1) composes a semi-continuous dynamic system (Ω, f, φ, M).

Definition 2.2. Let $f(P,t)$ be the semi-continuous dynamical system mapping described by system (2.1) at $\Omega \to \Omega$, $f(P,t)$ is a mapping in itself. If there are a point P_1 in phase set N and a t_1 such that $f(P_1,t_1) = Q_1 \in M\{x,y\}$, it also has $\varphi(Q_1) = \varphi(f(P_1,t_1)) = P_1 \in N$, then $f(P_1,t_1)$ is said to be the order-1 periodic solution.

Definition 2.3 [12]. Suppose N is the phase set of system (1.1), M is the impulse set of system (1.1), both N and M are straight lines (see Fig. 1). The intersection point of N and x axis is Q, the distance between point A ($A \in N$) and point Q is noted by a, M_1 denotes the intersection point of trajectory passing trough point A and M, phase point of M_1 is A_1 ($A_1 \in N$), the distance between A_1 and Q is noted by a_1. We define subsequent point of A is A_1, successor function of A is $f(A) = a_1 - a$.

Remark 2.1. If $f(A) = 0$, the trajectory passing through point A is the order-1 periodic solution of the system.

Lemma 2.1 [12]. *Successor function $f(A)$ is continuous.*

According to Lemma 2.1, we can get the following lemma.

Lemma 2.2 [12]. *Assume continuous dynamical system (X, Ψ), if there exist two points A, B in the phase set such that successor function $f(A) > 0, f(B) < 0$, we can find a point C between A and B in the phase set satisfying $f(C) = 0$. So there must exists an order-1 periodic solution passing through point C.*