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First-passage time statistics in a bistable system subject to Poisson white noise by the generalized cell mapping method

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ABSTRACT

The first-passage time statistics in a bistable system subject to Poisson white noise is studied by using the generalized cell mapping method. Specifically, an approximate solution for the first-passage time statistics in a second-order bistable system is developed by analyzing the motions in double-well potential and the global dynamics in phase space. Both symmetric and asymmetric cases have been investigated, and the effects of noise intensity and mean arrival rate of impulse on the first-passage time statistics are discussed respectively. It shows that the effect of Poisson white noise excitation on the first-passage time is quite different from that of the Gaussian one. With the same noise intensity, Poisson white noise can make for a faster first-passage.

1. Introduction

The first-passage time statistics is an important research topic in the reliability theory of stochastic dynamical systems. In this problem, we are interested in when the system response crosses the boundary of safe domain for the first time, in other words, the computation of the probability density function (PDF) of first-passage time or the mean first-passage time (MFPT). Many research works on the first-passage time for random walks [1,2] and the MFPT in bistable systems [3,4] have been carried out, but most of them are first-order cases. In second-order and above systems, the exact solutions of the first-passage time are usually not available. Hence, many approximate or numerical methods have been developed to solve the problem, such as energy envelope approximation method [5], the Galerkin technique [6,7], stochastic averaging method [8–10], general finite element solution method [11], cell mapping method [12], path integral method [13,14], and advanced Monte Carlo simulation method [15].

Poisson white noise is a versatile physical model for random pulse chain discretely distributed on the time axis. It has been widely adopted and studied in many fields of engineering, physics, biology and economics since it can describe the perturbances of discrete incidents more suitably than Gaussian white noise. The dynamical behaviors of nonlinear systems under Poisson white noise excitations have been investigated extensively in the past few decades. Among them, the study of first-passage problem [16–18] is quite attractive, because it reveals the unique non-Gaussian phenomena in the reliability study of systems in present of Poisson white noise. Hence, more attentions should be paid to the first-passage time statistics in second-order bistable systems subject to Poisson white noise excitations.

The generalized cell mapping (GCM) method is firstly proposed by Hsu [19] in 1981. As a new version of cell mapping method, it is not only effectively used to analyze the global properties and global bifurcations of nonlinear systems

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[20–24], but also a powerful tool to study the response analysis and stochastic bifurcation [25–29] of systems with random excitations. In Ref. [12], Sun and Hsu developed a procedure to obtain the first-passage time probability of nonlinear stochastic systems by the GCM method. It is superior to most of the analytical approximate methods because it is free of certain conditions on the system, such as light damping and weak non-linearity. Recently, based on the GCM method, some stochastic responses analysis of systems subjected to Poisson white noise excitations have been studied by Wu and Zhu [30] and Yue et al. [31]. These results indicate that the GCM method is effective to deal with the systems excited by Poisson white noise.

In the present paper, the first-passage time statistics in a bistable system driven by Poisson white noise is studied by a numerical approach based on the GCM method. First, the problem of first-passage in a second-order bistable system is introduced. Then, the GCM method is applied to obtain the approximate solutions of the first-passage problem in the bistable system. After that, the results of the first-passage time statistics and some relevant discussions are presented, and direct Monte Carlo simulation is used to confirm the effectiveness of the GCM method. Finally, the paper ends with some conclusions.

2. First-passage problem in a second-order bistable system

The physical significance of potential function is the potential energy of a unit mass. Consider a general double-well potential $V(x)$ given by the following polynomial

$$V(x) = -\frac{1}{2}x^2 - \frac{1}{3}\mu x^3 + \frac{1}{4}\alpha x^4, \tag{1}$$

in which $\mu \geqslant 0$, and it can be considered as the symmetry parameter. As is shown in Fig. 1, the double-well potential is symmetric when $\mu = 0$, and it is asymmetric when $\mu > 0$.

If a particle is moving in this potential driven by multiplicative Poisson white noise, we can obtain a second-order bistable dynamical system, which is described by the following stochastic differential equation

$$\ddot{x} + \zeta\dot{x} + \frac{\mathrm{d}V(x)}{\mathrm{d}x} = x\eta(t), \tag{2}$$

where ζ is the damping coefficient, and it equals to 0.42 in this paper. $\eta(t)$ represents the Poisson white noise [18,30], which has two important parameters, one of them is the noise intensity I, and the other one is the mean arrival rate λ, denoting the mean number of impulses per unit time.

When the system (2) is noise free ($I = 0$), the method proposed in Ref. [32] is used to analyzed the global properties. The results for different μ are presented in Fig. 2(a) and (b). The two periodic attractors of system $\mathbf{x_1}$ and $\mathbf{x_2}$ respectively represent the stable states at the bottom of the two potential wells. The corresponding basins of attraction are donated by $\boldsymbol{B_1}$ and $\boldsymbol{B_2}$. Besides, the basin boundary Γ and saddle $\boldsymbol{S}$ located at the origin can be considered as the potential barrier.

Then, we consider the first-passage problem in this bistable system when the noise intensity $I > 0$. In the study of transient probability density, it is found that probability density of response can evolve from one attractor to the other one along the unstable manifolds *UM* (see Fig. 2). So the direction of probability diffusion is mainly consistent with *UM*. Based on the global dynamics in the phase space, the basins of attraction $\boldsymbol{B_1}$ and $\boldsymbol{B_2}$ are regarded as the safe domains of stable states $\mathbf{x_1}$ and $\mathbf{x_2}$, respectively. And it can be stated that first-passage occurs when the response of system initial from one stable state crosses boundary Γ and reaches the safe domain of the other stable state for the first time.

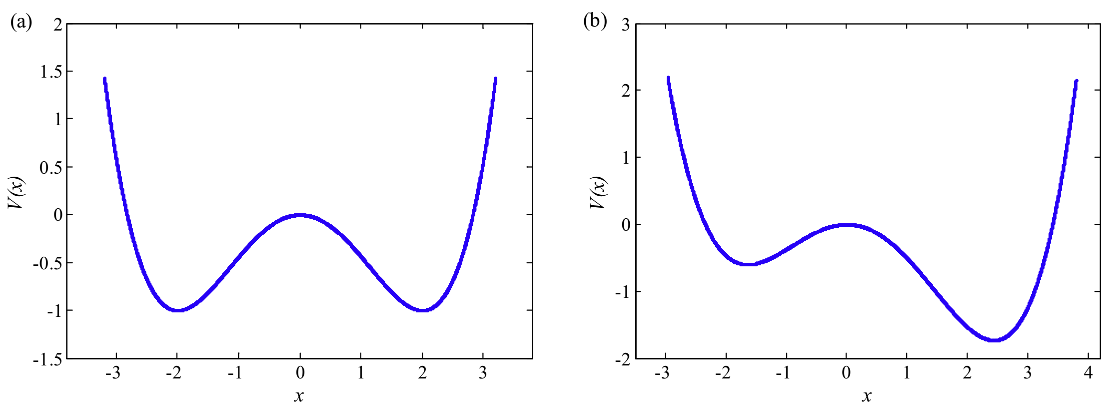

Fig. 1. The potential functions $V(x)$ when $\alpha = 0.25$, (a) $\mu = 0$, (b) $\mu = 0.2$.