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Arctic melt ponds and bifurcations in the climate system

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ABSTRACT

Understanding how sea ice melts is critical to climate projections. In the Arctic, melt ponds that develop on the surface of sea ice floes during the late spring and summer largely determine their albedo – a key parameter in climate modeling. Here we explore the possibility of a conceptual sea ice climate model passing through a bifurcation point – an irreversible critical threshold as the system warms, by incorporating geometric information about melt pond evolution. This study is based on a bifurcation analysis of the energy balance climate model with ice-albedo feedback as the key mechanism driving the system to bifurcation points.

1. Introduction

Sea ice is not only a sensitive, leading indicator of climate change, it is a key player in Earth's climate system. It also serves as a primary habitat for algal and bacterial communities which sustain life in the polar oceans. Perhaps the most visible, large scale change on Earth's surface in recent decades has been the precipitous decline of summer Arctic sea ice. With this significant loss of a white reflecting surface covering the Arctic Ocean, its albedo or reflectance decreases, and solar radiation is absorbed by the ocean rather than being reflected. This heats the upper ocean, melting even more ice, and so on, which is known as "ice-albedo feedback".

While global climate models predict a general decline in Arctic sea ice over the 21st century, the observed losses have significantly out-paced projections [19,26]. Improving our predictive capability for the fate of Earth's sea ice cover and its ecosystems depends on a better understanding of important processes and feedback mechanisms. For example, during the melt season the Arctic sea ice cover becomes a complex, evolving mosaic of ice, melt ponds, and open water. The albedo of sea ice floes is determined by melt pond configurations [20,22,24]. As ponds develop, *ice-albedo feedback* enhances the melting process. Understanding such mechanisms and their impact on sea ice evolution and its role in the climate system is critical to advancing how sea ice is treated in climate models and improving projections.

Conceptual, or *low order* climate models often introduce feedback through empirical parameterization, for example, taking into account a simple relation between temperature and area of ice covered surface. There is a wide range of such works, including [7,9,12,18]. Usually, ice-albedo feedback was simply associated with a decrease in ice covered area and a corresponding increase in the surface temperature, further decreasing the ice covered area. Given the key role that melt pond formation and evolution plays in sea ice albedo, we note here an apparent lack of incorporation of such features into conceptual models of ice-albedo feedback. Here we note that it is important to explore how melt pond geometry and thermodynamics affect conceptual climate models, and ice-albedo feedback in particular.

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While melt ponds form a key component of the Arctic marine environment, comprehensive observations or theories of their formation, coverage, and evolution remain relatively sparse. Available observations of melt ponds show that their areal coverage is highly variable, particularly for first year ice early in the melt season, with rates of change as high as 35 percent per day [22]. Such variability, as well as the influence of many competing factors controlling melt pond and ice floe evolution, make realistic treatments of ice-albedo feedback in climate models quite challenging [22]. Small and medium scale models of melt ponds which include some of these mechanisms have been developed [24,25], and melt pond parameterizations are being incorporated into global climate models [19].

Moreover, recently it has been found [14] that melt pond geometry has a complex fractal structure, and that the fractal dimension exhibits a transition from 1 to about 2 around a critical length scale of 100 m^2 in area. This behavior should be taken into account in investigating sea ice-albedo feedback.

Given the complex, highly nonlinear character of the underlying differential equations describing climate, it is natural to ask whether the decline of summer Arctic sea ice has passed through a so-called *tipping point*, or irreversible critical threshold as the system progresses toward ice-free summers [1,9]. A key mechanism potentially driving the system to "tip" is ice-albedo feedback. The main aim of this work is to investigate such a tipping point for a simplified model of sea ice and the climate system which takes into some account the evolution of melt pond geometry and its effect on sea ice albedo.

The surface of an ice floe is viewed here as a two phase composite of dark melt ponds and white snow or ice. The onset of ponding and the rapid increase in coverage beyond the initial threshold is similar to critical phenomena in the theory of phase transitions. Here we ask if the evolution of melt pond geometry – and sea ice albedo – exhibit universal characteristics which do not necessarily depend on the details of the driving mechanisms in numerical melt pond models. Fundamentally, the melting of Arctic sea ice is a phase transition phenomenon, where a solid turns to liquid, albeit on large regional scales and over a period of time which depends on environmental forcing and other factors. We thus look for features which are mathematically analogous to related phenomena in the theories of phase transitions and composite materials.

Basing our approach on the standard nonlinear phase transition model in the 2D case [6], we propose an expression for the rate of change of the melt pond size. It can be extended to the 3D case taking into account the vertical transfer of water to the ocean through ice due to the different physical processes. After that, we introduce the expression for albedo of the ice-covered surface and investigate through the melt pond size how the unexpected fractal geometry of melt ponds [14] can influence the formula for albedo of the ice covered surface.

As the next step, we consider a standard conceptual climate model– an ordinary differential equation (ODE) [12] with ice-albedo feedback taking into account the albedo of melt ponds. We modify this model assuming a stochastic distribution of melt pond sizes, based on the Fokker–Planck equation. After that we investigate equilibria of the resultant stochastic ODE under the key assumption that the surface temperature is a slow function of time relative to melt pond size. Different bifurcation regimes were obtained for this model. One of them may be quite interesting for climate applications, where the temperature of this system is stabilized only due to the fractal transition in melt pond geometry.

2. Evolution of melt ponds

2.1. Mechanism of the fractal transition

Viewed from high above, the sea ice surface can be thought of as a two phase composite of ice and melt water. The boundaries between the two phases evolve with increasing complexity and a rapid onset of large scale connectivity, or percolation of the melt phase (Fig. 1). As was shown in [14] that the melt pond perimeter Π can be defined approximately by

$$\Pi \sim \sqrt{S}^{D}, \tag{1}$$

here S is the area of ponds and D is the fractal dimension. The authors have observed a transition from $D = 1$ to $D \approx 2$ as the ponds grow in size, with the transitional regime centered around 100 m^2. According to [14] there exist three regimes:

A) $S < 10$ m^2; then we observe simple ponds with smooth boundaries and $D \approx 1$;
B) 10 $m^2 < S < 1000$ m^2; corresponding to transitional ponds where complexity increases rapidly with size;
C) $S > 1000$ m^2; complex, self-similar case, where pond boundaries behave like space filling curves with $D \approx 2$ (so-called fractals).

Here, one can show the transition in empirical formula (1) can be obtained from the rigorous pattern formation theory that uses the Kuramoto–Sivashinsky equation [16]. One can show that beginning with a critical characteristic size, the boundaries become unstable with respect to perturbations along the boundary.

We can suppose pond boundaries with fractal dimension about one can be considered like growing elliptical curves (there are circular ponds, in the ideal case) which become unstable at some characteristic size R, the lengths of the semi-major axe and the semi-minor one are $a_e = r_{e1}R$ and $b_e = r_{e2}R$, respectively, where a_e and b_e have the same order. In the case of fractal transition, the ponds are close to long and narrow ellipses, where a_e and b_e have the different order. These ellipses remind one of rivers rather than the simple circular ponds (Fig. 1). Then one can expect that the area of such a river of length R is proportional to R.