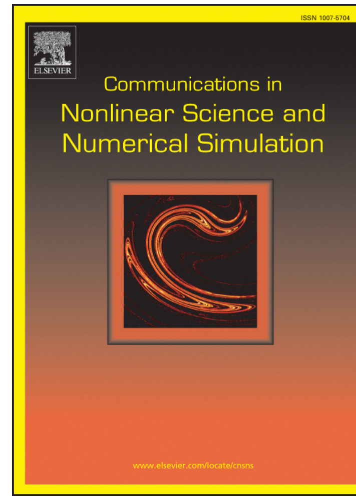

Short communication

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PII: S1007-5704(14)00355-4
DOI: http://dx.doi.org/10.1016/j.cnsns.2014.07.022
Reference: CNSNS 3289

To appear in: *Communications in Nonlinear Science and Numerical Simulation*

Received Date: 24 February 2014
Revised Date: 18 July 2014
Accepted Date: 25 July 2014

Counterexamples on Jumarie's two basic fractional calculus formulae

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August 5, 2014

Abstract

Juamrie proposed a modified Riemann-Liouville derivative definition and gave two basic fractional calculus formulae $(u(t)v(t))^{(\alpha)} = u^{(\alpha)}(t)v(t) + u(t)v^{(\alpha)}(t)$ and $(f(u(t)))^{(\alpha)} = f'_u u^{(\alpha)}(t)$. We give two counterexamples to show that Jumarie's two formulae are not true. Respectively, all results obtained in references by using Jumarie's these two formulae are incorrect. In the end, we give the corresponding formulae.

Keywords: counterexample, fractional calculus, modified Riemann-Liouville's derivative

1 Introduction

Jumarie proposed the following modified Riemann-Liouville fractional derivative[1-5]:

$$f^{(\alpha)}(t) = \frac{1}{\Gamma(1-\alpha)} \frac{\mathrm{d}}{\mathrm{d}t} \int_0^t (t-x)^{-\alpha}(f(x) - f(0))\mathrm{d}x. \tag{1}$$

and gave some basic fractional calculus formulae, for example, formulae (4.12) and (4.13) in [4]:

$$(u(t)v(t))^{(\alpha)} = u^{(\alpha)}(t)v(t) + u(t)v^{(\alpha)}(t), \tag{2}$$

$$(f(u(t)))^{(\alpha)} = f'_u u^{(\alpha)}(t). \tag{3}$$

The last formula (3) has been applied to solve the exact solutions to some nonlinear fractional order differential equations[6-9]. If this formula were true, then we could take the transformation $\xi = x - \frac{kt^\alpha}{\Gamma(1+\alpha)}$, and reduce the partial derivative $\frac{\partial^\alpha U(x,t)}{\partial t^\alpha}$ to $U'(\xi)$. Therefore the corresponding fractional differential equations become the ordinary differential equations which are easy to study.