Commun Nonlinear Sci Numer Simulat xxx (2014) xxx–xxx

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Delocalized periodic vibrations in nonlinear LC and LCR electrical chains

G.M. Chechin *, S.A. Shcherbinin

Department of Physics, Southern Federal University, Zorge 5, Rostov-on-Don 344090, Russia

ARTICLE INFO

Article history:
Received 3 July 2014
Received in revised form 27 September 2014
Accepted 29 September 2014
Available online xxxx

Keywords:
Nonlinear dynamics
Lattice models
Nonlinear normal modes
Invariant manifolds
Group-theoretical methods

ABSTRACT

We consider electrical LC- and LCR-chains consisting of N cells. In the LC-chain each cell contains a linear inductor L and a nonlinear capacitor C, while the cell in the LCR-chain include additionally a resistor R and an voltage source. It is assumed that voltage dependence of capacitors represents an even function. Such capacitors have implemented by some experimental groups studying propagation of electrical signals in the lines constructed on MOS and CMOS substrates. In these chains, we study dynamical regimes representing nonlinear normal modes (NNMs) by Rosenberg. We prove that maximum possible number of symmetry-determined NNMs which can be excited in the considered chains is equal to 5. The stability of these modes for different N is studied with the aid of the group-theoretical method [Physical Review E 73 (2006) 36216] which allows to simplify radically the variational systems appearing in the Floquet stability analysis. For NNMs in LC-chain, the scaling of the voltage stability threshold in the thermodynamic limit ($N \to \infty$) is determined. It is shown that the above group theoretical method can be also used for studying stability of NNMs in the LCR-chains.

1. Introduction

In recent years, studying of nonlinear vibrations in mesoscopic systems of different physical nature received much attention. In particular, discrete breathers and various soliton-like excitations were actively studied in cantilever arrays [1,2], granular crystals [3], Josephson junction lattices [4,5], photonic crystals [6], electrical lines [7–11], etc. It is essential that in contrast to crystals, where only indirect experiments are possible, in mesoscopic systems one can often observe different dynamical objects directly.

In this paper, two families of electrical lines are considered. We call them LC- and LCR-chains. The former consist of nonlinear (voltage-dependent) capacitors C_j coupled by linear inductors L_j, while the latter include also resistors R_j, as well as harmonic external voltage sources $U_j \sin(\Omega t)$. These chains are depicted schematically in Fig. 1 and 2, respectively. They are formed by $j = 1 \ldots N$ electrical cells consisting of the above mentioned elements L_j, C_j, R_j and $U_j \cos \Omega t$.

We consider specific dynamical regimes, which represent *nonlinear normal modes* (NNMs) by Rosenberg [12] in LC- and LCR-chains. These regimes determined the time-dependent distribution of voltages $V_j(t)$ on capacitors and currents $I_j(t)$ through inductors and resistors. The number of the chain cells (N) must be different to support different NNMs.

* Corresponding author.
E-mail address: gchechin@gmail.com (G.M. Chechin).

http://dx.doi.org/10.1016/j.cnsns.2014.09.028
1007-5704/© 2014 Elsevier B.V. All rights reserved.

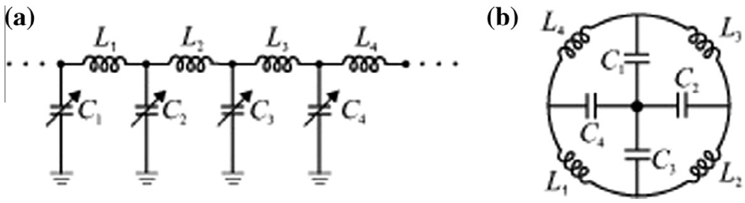

Fig. 1. Chain of nonlinear capacitors coupled by linear inductors (LC-chain).

Hereafter we assume that the dependence $C(V)$ of the capacity C on the voltage V represents an even function $C(-V) = C(V)$. Such capacitors have implemented by some experimental groups [13–16] who studied propagation properties of various electrical signals in the lines constructed on MOS and CMOS substrates. Following the theoretical paper [10], we assume that the function $C(V)$ possesses the form

$$C(V) = C_0(1 - bV^2), \tag{1}$$

where C_0 and b are positive constants[1].

We study only *symmetry-determined* NNMs (SD NNMs), i.e. those modes which are allowed by symmetry principles.

The simplest nonlinear normal mode represents the so-called zone boundary mode, or π-mode. The existence and stability of this mode in the LC-chain was studied in Ref. [10]. It can be written in the form

$$\phi_1 = \{V(t), -V(t)|V(t), -V(t)|\dots\} = V(t)\{1, -1|1, -1|\dots\}. \tag{2}$$

This means that voltages on every pair of neighboring capacitors are opposite in sign at any time. Obviously, such dynamical regime can exist only in the chains with an even number of cells.

There are many papers devoted to study π-mode in monoatomic chains of the Fermi–Pasta–Ulam (FPU) type [17–23]. However, the LC-chain model essentially differs from that of FPU. In Appendix of Ref. [10], the authors reduced equations of the LC-chain to those of the FPU-β model using some approximations. However, we prefer to study the stability of π-mode, as well as all other NNMs, for the exact dynamical LC-equations [see Eq. (5) below].

All NNMs represent time-periodic regimes and, therefore, their stability can be analyzed with the aid of the standard Floquet method. The dimension of the variational system describing equations linearized in a vicinity of the given NNM and the dimension of the corresponding monodromic matrix is equal to $2N$, where N is the number of the chain cells. Because of this reason, the studying of the linear stability of NNMs turns out to be very difficult in the case $N \gg 1$, especially when $N \to \infty$. A specific method for analyzing the π-mode stability was developed in [10]. However, this method is based essentially on the concrete structure of the dynamical equations of the LC-chain.

On the other hand, a general group-theoretical method for splitting (decomposing) variational system into *independent subsystems*, whose dimensions can be considerably smaller than that of the original variational system, was developed for arbitrary dynamical models with discrete symmetry in our papers [23,24]. This method is based only on some symmetry-related principles and it uses the apparatus of irreducible representations of the symmetry group of the considered dynamical regime. In Refs. [20,21], we used this method for analyzing linear stability of all possible symmetry-determined Rosenberg nonlinear normal modes in the FPU-α and FPU-β chains. In Refs. [25,26], it was applied for studying stability of discrete breathers and quasibreathers in $2D$ scalar dynamical models on the plane square lattices.

In the present paper, we use the above group-theoretical method for analyzing linear stability of the symmetry-determined nonlinear normal modes in the electrical chains of LC and LCR types.

The paper is organized as follows. In Section 2, we consider the explicit form of the dynamical equations describing LC- and LCR-chains. The possible symmetry-determined nonlinear normal modes (NNMs) in the LC-model are discussed in Section 3, while the group-theoretical method for studying their stability is briefly reviewed in Section 4. We apply this method for splitting variational systems for all NNMs permissible in the above model in Section 5. The next section is devoted to study nonlinear normal modes and their stability in the LCR-chains. In Conclusion, we summarize the results of this paper and discuss perspectives of our group-theoretical approach to study existence and stability of quasiperiodic nonlinear vibrations (bushes of NNMs) in the electrical chains.

2. LC- and LCR-models

Applying the Kirchhoff laws for quasi-stationary current to the circuit in Fig. 1, one can obtain the following equations (see Ref. [10]):

$$L_j \frac{dI_j}{dt} = V_j - V_{j+1}, \quad \frac{dQ(V_j)}{dt} = I_{j-1} - I_j. \tag{3}$$

[1] Note that our group-theoretical methods are suitable for any even $C(V)$. However, numerical results obtained in this paper correspond just to $C(V)$ determined by Eq. (1).