Commun Nonlinear Sci Numer Simulat xxx (2014) xxx–xxx

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Vibrational resonance in a heterogeneous scale free network of neurons

Muhammet Uzuntarla [a,*], Ergin Yilmaz [a], Alexandre Wagemakers [b], Mahmut Ozer [c]

[a] Department of Biomedical Engineering, Engineering Faculty, Bülent Ecevit University, 67100 Zonguldak, Turkey
[b] Nonlinear Dynamics, Chaos and Complex Systems Group, Departamento de Física, Universidad Rey Juan Carlos, Tulipán s/n, 28933 Móstoles, Madrid, Spain
[c] Department of Electrical and Electronics Engineering, Engineering Faculty, Bülent Ecevit University, 67100 Zonguldak, Turkey

ARTICLE INFO

Article history:
Received 31 January 2014
Received in revised form 27 June 2014
Accepted 30 August 2014
Available online xxxx

Keywords:
Vibrational resonance
Diversity
Scale free networks

ABSTRACT

Vibrational resonance (VR) is a phenomenon whereby the response of some dynamical systems to a weak low-frequency signal can be maximized with the assistance of an optimal intensity of another high-frequency signal. In this paper, we study the VR in a heterogeneous neural system having a complex network topology. We consider a scale-free network of neurons where the heterogeneity is in the intrinsic excitability of the individual neurons. It is shown that emergence of VR in heterogeneous neuron population requires less energy than a homogeneous population. We also find that electrical coupling strength among neurons plays a key role in determining the weak signal processing capacity of the heterogeneous population. Lastly, we investigate the influence of interneuronal link density on the VR and demonstrate that the energy needed to obtain the resonance grows with the increase in average degree.

1. Introduction

The concept of resonance generally refers to an increase in the amplitude of the oscillations provoked by a particular external forcing or signal. In a nonlinear system when it is a high-frequency periodic signal that enhances the amplitude, the phenomenon is called vibrational resonance (VR) [1]. The role of resonances in different biological processes is capital, and recently attention has been paid to this phenomenon in biology [2–7].

Neural networks are biological units that process incoming sensorial or motor information, in form of electrical or chemical signals, with the purpose of taking a determinate action. The underlying dynamics of these signals can span over a variety of time scales: from milliseconds to days. One of the most significant example of existence of such bichromatic signals in the nervous system is the bursting neuron which operates in two widely different time scales [8]. Thus, it is not rare to have periodic signals with very different frequency at the input of a given neuronal network. As we will study in this article, the VR phenomenon is likely to occur in neuronal networks when two periodic signals with widely-separated frequency are present simultaneously at the input. In terms of information processing, the fast periodic forcing can help the network to detect and amplify a weak input signal. In this view, an interpretation of the VR is the optimization of the signal to noise ratio by the perturbation. In recent years, several works, though not many, have investigated the emergence of VR in neural systems both at the level of single neuron and networks with different topologies and coupling scheme [4,9–15]. A common assumption in these modeling studies is that the neurons in the population have been considered as identical units forming a

* Corresponding author.
E-mail address: muzuntarla@yahoo.com (M. Uzuntarla).

http://dx.doi.org/10.1016/j.cnsns.2014.08.040

Please cite this article in press as: Uzuntarla M et al. Vibrational resonance in a heterogeneous scale free network of neurons. Commun Nonlinear Sci Numer Simulat (2014), http://dx.doi.org/10.1016/j.cnsns.2014.08.040

homogeneous system. However, actual neuronal populations, even cells within the same functional column, exhibits a prominent heterogeneity in their response properties, such as mean firing rate, receptive field location and size, and stimulus selectivity [16]. It has been demonstrated both experimentally and theoretically that the heterogeneity in neural systems is relevant in many contexts, such as in synchronization phenomenon [17,18], coherence resonance [19], neural coding efficiency [16], reliability [20] or adaptation [21].

Our prime interest here is to study the importance of the diversity among neurons as to the collective detection of a low frequency signal by the network. We understand diversity as the spread of the intrinsic excitability among a population of neurons, which we will call a *heterogeneous population*. It is a natural assumption if we consider a biological system, as no one is expecting uniformity in a real network. The excitability controls the reactions of a neuron to a given input signal. Consequently, the global behavior of a heterogeneous network will differ from the homogeneous case. We will see in Section 3.2 how the diversity in the excitability is a key parameter regarding the VR.

Obviously, there are many more parameters affecting the possible occurrence of the VR in a neuronal network. Among others, the topology and coupling strength are particularly interesting since they dramatically affect the dynamics of the network. In Section 3.3 the coupling strength is treated as a global parameter that varies for all neurons. We find that this global coupling can optimize the detection of the input signal of the network.

In the last part of the study, we investigate the influence of the network structure on the VR. The network is constructed following a scale-free law of degree distribution, which is a topology that has been found in many fields of science including neurosciences [22]. This topology is used as a universal model through the article. We will discuss its relevance and its significance for the VR in Section 3.4.

2. Model and methods

We consider a population of non-identical FitzHugh–Nagumo (FHN) neurons that is governed by the following differential equations:

$$\varepsilon\frac{dv_i}{dt} = v_i - \frac{v_i^3}{3} - w_i + \sum_j g_{ij}(v_j - v_i), \tag{1}$$

$$\frac{dw_i}{dt} = v_i + a_i + I_{ex}, \tag{2}$$

where v_i and w_i represent the fast activation variable (membrane potential) and the slow recovery variable of neuron i, respectively. $\varepsilon = 0.01$ is the inherent time scale that separate the fast and slow dynamics. In this work, we assume that all the neurons in the population are subject to two different periodic signals injected to the neuron through the external current

$$I_{ex} = A\cos(\omega t) + B\cos(\Omega t + \varphi_i). \tag{3}$$

The information to be processed is encoded in the weak signal $A\cos(\omega t)$ having a low frequency ω and amplitude A. The other external drive $B\cos(\Omega t + \varphi_i)$ indicates a high frequency signal with amplitude B, frequency $\Omega \gg \omega$ and phase $\varphi_i \in [-\pi, \pi]$. Unless stated otherwise, we set the external signals parameters as $A = 0.01,\ \omega = 0.1$ and $\Omega = 5$ (dimensionless parameters).

In Eq. (2), the parameter a_i affects the dynamics of individual neurons, hence allowing us to control the diversity of the neuron population. The FHN neuron model is excitable having a stable fixed point for $a_i > 1$, and it exhibits oscillatory behavior for $a_i < 1$. When $a_i = 1$, the stability of the fixed point is lost through a Hopf bifurcation. Presently in our modeling, we allow a_i to take numerical values from a Gaussian probability distribution, satisfying $\langle a_i \rangle = a_0$ and $\langle (a_i - a_0)(a_j - a_0) \rangle = \delta_{ij}\sigma^2$. σ will be referred from now on as the diversity strength. When the firing threshold is normally distributed ($\sigma > 0$) we call the network heterogeneous. The homogeneous case ($\sigma = 0$) occurs when all the neurons are identical in parameters.

Since we are interested in the weak signal detection performance of the neurons in their excitable regime in presence of a high-frequency driving, we set $a_0 = 1.05$ that keeps all the neurons far away from the bifurcation point over certain interval of σ. Finally, g_{ij} is the bidirectional electrical coupling strength between neuron i and j, being $g_{ij} = g_{syn}$ if the two are connected and $g_{ij} = 0$ otherwise.

As the underlying interaction network of the neuron population, we use a scale-free (SF) topology based on the preferential attachment algorithm [23]. The construction process of SF network starts with a set of m fully connected nodes, and at each time step every new node makes m links to m different nodes already present in the network. To incorporate the preferential attachments of these links, we assume that the probability Π that a new vertex will be connected to node i depends on its degree k_i, such that $\Pi = k_i/\sum_j k_j$. After a time evolution, this procedure builds a network with an average degree $\langle k \rangle = 2m$, and a power-law degree distribution $P(k) \simeq k^{-3}$. Without loss of generality, we set $m = 2$ and $N = 200$ nodes throughout this work, yielding a SF network with an average degree $\langle k \rangle = 4$.

To evaluate the effect of VR on the network, the collective temporal behavior is measured by calculating the average membrane potential