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Pseudo Phase Plane and Fractional Calculus modeling of western global economic downturn

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ABSTRACT

This paper applies Pseudo Phase Plane (PPP) and Fractional Calculus (FC) mathematical tools for modeling world economies. A challenging global rivalry among the largest international economies began in the early 1970s, when the post-war prosperity declined. It went on, up to now. If some worrying threatens may exist actually in terms of possible ambitious military aggression, invasion, or hegemony, countries' PPP relative positions can tell something on the current global peaceful equilibrium. A global political downturn of the USA on global hegemony in favor of Asian partners is possible, but can still be not accomplished in the next decades. If the 1973 oil chock has represented the beginning of a long-run recession, the PPP analysis of the last four decades (1972–2012) does not conclude for other partners' global dominance (Russian, Brazil, Japan, and Germany) in reaching high degrees of similarity with the most developed world countries. The synergies of the proposed mathematical tools lead to a better understanding of the dynamics underlying world economies and point towards the estimation of future states based on the memory of each time series.

1. Introduction

This last 2007–2008 crisis was very severe, and many countries still drag on its consequences, suffering from high unemployment rates, sluggish economic recovery, and banking financial distress [1]. As far as those difficulties translated into social unrest and migration movements in the world scene, poor expectations among some analysts led them to inquire on the capacity to overcome these difficulties. Pessimistic views even question the Western world ability to preserve global hegemony [2], and frequently tell on an acceleration of convergence. From this perspective, Fogel 2007 presents the possibility of a real threat of a global geopolitical turnover by 2040. The relative weight of the USA GDP on the world GDP will decline from 22% to about 14%. In the same way, the relative weight of the European GDP on the world GDP will decline from 21% to about 5%. A successful convergence of Asian partners with core countries will make democracy depend on those Asian partners.

Europe and the United States of America are now much more optimistic in what respects GDP growth for the next years. However, economic historians have compared nowadays indicators with past crises indicators, and concluded that the current crisis has a new scale in comparison with past crises: In the (UK, for example) the "banking system has faced episodes of instability at various points throughout the past two centuries", but crises in the nineteenth century were much less severe

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as "instability increases significantly in the second half of the twentieth century, which is consistent with the idea that the frequency of financial instability has increased in this period" [3–6].

Of course crises are normal episodes. Capitalism is a flexible self-regulated system made of prosperity and crisis [7]. If prosperity and bubbles create crises and depressions, crises and depressions always have given place to new prosperity [8]. The end of capitalism prophecy based on Marxian views never materialized [9]. Capitalism resilience for survival results from business cycles that alternate prosperity and crisis.

At the same time, convergence is a well-studied issue, and deserves the humankind approval, for improving the lowest conditions of life on the face of the planet, contributing for global welfare and happiness. In 1960 the economic historian Walter Rostow analyzed the available evidence on convergence, and considered the different moments when industrialization took off in the past, for the different 1960s industrialized countries [9]. According to him, a common path exists for each country after the industrialization take-off. Investment and new technologies leads any industrialization process to maturation, and each one of these partners would get a steady-state phase of large and generalized mass consumption patterns. Seventy years were enough to close the distance from take-off to a steady state mass consumption.

The Multi-dimensional scaling method was applied to the analysis of economic growth have proved that convergence stands mainly among European partners, some European offshores (Canada, USA, and Australia), and Japan [10,11]. Asian partners such as India and China still remain far from a convergence with core countries.

In this paper these questions are approached in using Pseudo Phase Plane (PPP) and Fractional Calculus (FC) tools to discuss the global dynamics of the world system and convergence of ten major countries, in order to check both views. The PPP method allows the study of the dynamics of time series while avoiding the numerical calculation of derivatives. On the other hand, FC is a mathematical tool that embeds the modeling of phenomena with long term memory effects. Therefore, the common adoption of the two methodologies allows not only a direct visualization and interpretation of the dynamics of country economies, but also the formulation of schemes for estimating the future evolution of the time series. The ten selected major global partners are the European UK, Germany, and Russia, the Asian China, India, and Japan, and the American USA and Brazil. Having these ideas in mind, the paper is organized as follows. Section 2 presents the fundamentals of pseudo phase space representation and fractional operators. Section 3 is devoted to methodology and analytical aspects of economic dynamics. Section 4 presents the estimations and discusses their accuracy. Finally, Section 5 outlines the main conclusions.

2. Preliminary concepts

This section introduces the main concepts that motivate this study. Section 2.1 describes the PPP method and Section 2.2 presents the fundamentals of FC.

2.1. Pseudo Phase Plane

The Pseudo Phase Space (PPS) is a technique useful for analysing systems with complex dynamics where complete information about all states is not available. In comparison with the classical phase space method, the PPS method is more robust to signal noise and allows the representation in a higher dimensional space by using a small sample of measurements of the system time history. The PPS reconstruction is based on Takens' embedding theorem [12]. If a signal $x(t)$ is one component of an attractor that can be represented by a smooth d-dimensional manifold, then the topological properties of the time series are equivalent to those of the embedding formed by the n-dimensional vectors:

$$v(t) = [x(t)\ x(t+\tau)\ x(t+2\tau)\ \cdots\ x[t+(n-1)\tau]] \tag{1}$$

where $n > 2d+1,\ d, n \in \mathbb{N}$ and $\tau > 0$. The parameters and n denote the time delay and the embedding dimension, respectively. The vector $v(t)$ is often depicted in a n-dimensional PPS graph that depends on the values of τ and n. Usually, it is adopted $n = 2$ or $n = 3$ since they allow a direct visualization and interpretation.

In the case of $n = 2$ the PPS degenerates into the PPP. Therefore, the PPP vector $[x(t)\ x(t+\tau)]$ is related to the classical phase plane vector $[x(t)\ \dot{x}(t)]$ and allows the study of the system dynamics.

An important aspect of the PPP and PPS methods consists of the calculation of the time delay τ. Let us adopt a practical point of view and consider that the signal $x(t)$ is not extremely noisy. If we have a small value of τ, then the series $x(t)$ and $x(t+\tau)$ are very close (they do not provide two independent coordinates) and we get simply a 45° line in the PPP chart. On the other hand, if τ is very large, then the two series are almost independent (yielding unrelated coordinates), but the common intersection of pairs of values vanishes. In fact, the larger the value of n the smaller the intersection since for a time series of finite length L the intersection has length $L-(n-1)\tau$. Therefore, some practical compromise is needed for choosing the "optimal" time delay, τ_m that minimizes some kind of index $J[v(t)]$:

$$\min_{\tau} \{J[v(t)]\} \tag{2}$$