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Nonlinear dynamic analysis and characteristics diagnosis of seasonally perturbed predator–prey systems

Huayong Zhang [a], Tousheng Huang [a,b], Liming Dai [b,*]

[a] Research Center for Ecological Engineering and Nonlinear Science, North China Electric Power University, Beijing 102206, PR China
[b] Industrial Systems Engineering, University of Regina, Regina, SK S4S 0A2, Canada

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ABSTRACT

Predator–prey interaction widely exists in nature and the research on predator–prey systems is an important field in ecology. The nonlinear dynamic characteristics of a seasonally perturbed predator–prey system are studied in this research. To study the nonlinear characteristics affected by a wide variety of system parameters, the PR approach is employed and periodic, quasiperiodic, chaotic behaviors and the behaviors between period and quasiperiod are found in the system. Periodic–quasiperiodic–chaotic region diagrams are generated for analyzing the global characteristics of the predator–prey system with desired ranges of system parameters. The ecological significances of the dynamical characteristics are discussed and compared with the theoretical research results existing in the literature. The approach of this research demonstrates effectiveness and efficiency of PR method in analyzing the complex dynamical characteristics of nonlinear ecological systems.

1. Introduction

Predator–prey interaction is a basic relationship in nature. The predator–prey relationship may show oscillatory behaviors and take the forms of resource-consumer, plant–herbivore, phytoplankton–zooplankton, and so on [7]. Due to the importance of predator–prey interaction, the dynamical relationship between predator and prey has been extensively studied and becomes one of the dominant themes in population ecology [2,13].

In the pioneering works of Lotka and Volterra, a predator–prey model was proposed [10,21]. The Lotka–Volterra predator–prey model is probably the simplest and most basic predator–prey model in the field. On the basis of the Lotka–Volterra model, a great number of dynamical models have been developed to study the predator–prey interaction from various aspects [18,9,11]. The development may be alteration of the model functions (e.g. the Holling types I, II, III classification) to describe different ecological processes in the predator–prey interaction [19,17].

Several researchers argued that non-autonomous model was closer to realistic predator–prey system than autonomous model [11]. On this basis, the non-autonomous models were also developed to study the dynamical behaviors of the predator–prey system. For example, predator–prey models with time delay were studied because the predator–prey interaction may have time lag [22]. In order to incorporating the influence of seasonal variation on the predator–prey interaction, predator–prey models with periodically varying parameters were also proposed [20,6].

The research works existing in literature demonstrated that the seasonally perturbed predator–prey systems may have abundant nonlinear dynamical behaviors, such as multiplicity of attractors, quasiperiodic behavior, and deterministic chaos

* Corresponding author.

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[14,15,6]. In analyzing the periodic and non-periodic behaviors of the predator–prey system, Lyapunov exponent and Poincaré map are often employed [8,6]. Moreover, a periodicity ratio (PR) method was developed [3,4] to conveniently diagnose a dynamical system for its chaotic and periodic behavior. The PR method shows effectiveness in quantitatively describing the dynamical behaviors of nonlinear systems.

One of the seasonally perturbed two species predator–prey systems was studied by Beckers and Nihoul [1] with the model established by the authors. The two species predator–prey system studied by the authors is representative. Compared with the autonomous predator–prey model, the model considered seasonal variations shows advantages in analyzing the predator–prey relationship and the dynamical behaviors of the ecological system. In diagnosing the characteristics of the system the authors employed graphics such as Poincaré maps. With the approach of the authors, chaotic behavior was shown, though the result of chaos may be not necessarily correct.

In this research, the characteristics of the seasonally perturbed predator–prey system are to be further studied with a new approach. The characteristics of the system are to be diagnosed with the employment of the PR method. Focus will be laid upon systematic and quantitative analyses of the nonlinear dynamic characteristics of the predator–prey system, especially the characteristics that were not described or incorrectly described by the previous researchers including Beckers and Nihoul [1]. Ecological significances corresponding to the nonlinear dynamic characteristics of the predator–prey system are to be described. With the approach of this research, detailed and more accurate results will be developed with the system parameters of much large range. The results of the present research may provide guidance for analyzing and characterizing nonlinear predator–prey systems.

The structure of this research is arranged as follows. Section 2 describes the seasonally perturbed predator–prey system, and previous analysis made by Beckers and Nihoul [1]. Section 3 introduces the PR method. Section 4 demonstrates the characteristics of the seasonally perturbed predator–prey system via numerical simulations. Section 5 gives the discussion on the results obtained. The conclusion of this research is to be presented in Section 6.

2. Seasonally perturbed predator–prey system

2.1. System description

The predator–prey interaction is usually imbedded in periodically varying environments. The periodic variation in ecological systems may result from change of season. For example, variation of precipitation greatly affects biomass growth rate of vegetation in arid and semiarid dryland during a year, light intensity controls photosynthesis during the seasons [15]. Scheffer et al. [16] described that seasonal cycle of temperature and light and fish predation imposes strongly influence on the predator–prey interaction between algae and zooplankton. The study on seasonally perturbed predator–prey system has realistic ecological significance and the effect of seasonal variation on predator–prey interaction can be identified. On the other hand, the abundant dynamical behaviors of seasonally perturbed predator–prey system can be used to interpret the complexity of the population dynamics.

The seasonally perturbed predator–prey model studied in this research can be described by the following governing equations [1]:

$$\frac{dy_1}{dT} = a(T)y_1 - by_1y_2, \tag{1}$$

$$\frac{dy_2}{dT} = cy_1y_2 - dy_2, \tag{2}$$

in which y_1 is density of the prey, y_2 is the density of the predator, T is time. $a(T)$ is the intrinsic growth rate of the prey, determined by the difference between basic birth and death rates of the prey; b is a constant predation rate (fraction of the prey population eaten per predator), so that total predation is proportional to the abundance of prey and the abundance of predator. c/b is a constant conversion rate of eaten prey into new predator abundance; d is a constant per capita mortality rate of predator. Moreover, the intrinsic growth rate of the prey, $a(T)$, is periodically varying due to seasonal variation, and described by the following:

$$a(T) = a_0 + a_1 \sin(wT), \tag{3}$$

Table 1
Nonlinear dynamical behaviors of the predator–prey system shown in figures.

Figure number	Parameter set			PR value	Dynamical behavior
	r	ε	n		
Fig. 2	0.95	0.1	1	1	Period
Fig. 3	0.9	0.1	0.9	0	Quasiperiod
Fig. 4	0.9	0.2	1.78	0	Chaos
Fig. 7	0.9	0.1	1.99	0.8751	Behaviors in between period and quasiperiod
	0.9	0.2	2.71	0.2218	