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Linear stability of a generalized multi-anticipative car following model with time delays

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ABSTRACT

In traffic flow, the multi-anticipative driving behavior describes the reaction of a vehicle to the driving behavior of many vehicles in front where as the time delay is defined as a physiological parameter reflecting the period of time between perceiving a stimulus of leading vehicles and performing a relevant action such as acceleration or deceleration. A lot of effort has been undertaken to understand the effects of either multi-anticipative driving behavior or time delays on traffic flow dynamics. This paper is a first attempt to analytically investigate the dynamics of a generalized class of car-following models with multi-anticipative driving behavior and different time delays associated with such multi-anticipations. To this end, this paper puts forwards to deriving the (long-wavelength) linear stability condition of such a car-following model and study how the combination of different choices of multi-anticipations and time delays affects the instabilities of traffic flow with respect to a small perturbation. It is found that the effect of delays and multi-anticipations are model-dependent, that is, the destabilization effect of delays is suppressed by the stabilization effect of multi-anticipations. Moreover, the weight factor reflecting the distribution of the driver's sensing to the relative gaps of leading vehicles is less sensitive to the linear stability condition of traffic flow than the weight factor for the relative speed of those leading vehicles.

1. Introduction

Traffic flow theory has been developed since 1950s, for example the simple kinematic wave model of [15,24], to understand and analyze many complex traffic phenomena including the transitions between many congested traffic states as described in [25]. Depending on the level of detail, traffic flow models are classified in either microscopic models or macroscopic models. Microscopic models describe the dynamics of traffic flow at the high level of detail such as the movement of individual vehicles, for example car-following models [30,11,4] while the macroscopic models represent traffic flow at low level of detail by aggregate traffic variables such as flow, mean speed and density [9,35,37,5,27,21,14,29,38,26,28]. Recently, there have been recent impressive advances in modeling the dynamics of traffic flow which realistically consider the drivers' reaction to the behavior of many vehicles ahead (i.e. multi-anticipative driving behavior). This driving behavior has been investigated both numerically and analytically using either microscopic models [13,6,3,32,7,8,10,12] or macroscopic models [33,16,17,22]. At the microscopic level, the multi-anticipative driving behavior has been studied using either extended

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Intelligent Driver models-IDM [32,10,12], extended optimal velocity model-OVM [13,6,33,3,36] or other type of car-following models [7,8] while the second order macroscopic model type is used to analyze the dynamics of the multi-anticipative traffic flow [33,18,22]. Another line of research focuses on taking delay or reaction time into account. By definition, the delay is a physiological parameter reflecting the period of time between perceiving a stimulus and performing a relevant action. The effect of such delay on traffic flow dynamics has also been investigated both analytically and numerically in traffic flow literature [1,32,21,19,20] and many references there-in.

In general, most current theoretical studies do not consider the combined effect of multi-anticipations and different delays associated with such multi-anticipations. To contribute to the development of traffic flow theory, this paper attempts to derive the analytical condition reflecting the combined effect of both multi-anticipations and delays on traffic flow dynamics using a generalized car-following model. Basically, the generalized car-following model includes the dependence of the optimal speed function on the gap and the relative speed of the driver with leading vehicle(s). Such a dependency has been reported to represent vehicle interactions more realistically which avoid accidents in some certain situations and has changed the stability conditions significantly than a simple dependence of the optimal speed headway on the gap as the OVM, which will be shown in the ensuing paper. To this end, the contribution of this paper is twofold:

1. The generalized car-following model is extended to capture the multi-anticipative microscopic driving behavior with delays.
2. The analytical properties of the model are derived to show the general (long-wavelength) linear conditions influencing the instabilities of traffic flow.

This paper is organized as follows. Section 2 introduces an extended generalized car-following model for multi-anticipative driving behavior with delays. Section 3 shows the properties of the introduced model in which the linear stability method is used to derive the general condition influencing the instabilities of traffic flow. Finally, we conclude the paper in Section 4.

2. Model equation

Multi-anticipative car-following models describe the motion of individual vehicle n in relation to its leading vehicles $(n-m+1)$ where $m = 1,2,3\ldots,M$. In this work, we will use (single-lane) time-continuous model, where the acceleration of vehicle n is of the general form:

$$\frac{dv_n(t)}{dt} = f\left(v_n(t), \sum_m \alpha_m s_{n-m+1}(t-\tau^s_{n-m+1}), \sum_m \beta_m \Delta v_{n-m+1}(t-\tau^\Delta_{n-m+1})\right), \quad (1)$$

Eq. (1) represents a generalized multi-class car-following model which allows for the dependence of the optimal speed function on the gap and the relative speed between the considered vehicle and many vehicles ahead. In this model, time delays vary in sensing relative headway (i.e. τ^s_{n-m+1}) and relative speeds (i.e. τ^Δ_{n-m+1}) with many vehicles ahead. In principle, we neglect the delay with respect to the speed of the considered vehicle because drivers are aware of their own speed immediately [23].

In Eq. (1), α_m and $\beta_m (m \in M)$ represents the weight factors for the relative gap and relative speed which generally satisfies $\alpha_1 > \alpha_2 > \ldots > \alpha_M$, $\beta_1 > \beta_2 > \ldots > \beta_M$ and $\sum_m \alpha_m = 1, \sum_m \beta_m = 1$. Basically, M is related to the traffic conditions as well as the layout of the road. Recent empirical study by [7] based on data collected at a small section of a freeway in the Netherlands has indicated that $M = 3$ is a reasonable value. However, this is just an indication for a certain freeway section during a certain period of congested traffic. We think there should be more extensive data to be studied in order to get a good conclusion. In this paper, we will show how the choice of α_m and β_m influences the linear traffic instabilities.

Notation.
Index

x_n	Location of vehicle n (m)
t	Time instant (s)
m	Number of leading vehicles $(m = 1,2,\ldots,M)$

Model variables

v_n	Speed of vehicle n (m/s)
Δv_{n-m+1}	Relative speed of vehicle $n-m+1$ and its leader $(n-m)$ (m/s)
s_{n-m+1}	Bumper-to-bumper gap between vehicle $n-m+1$ and its leaders $(n-m)$ (m)

Model parameters

v^0	Desired speed (m/s)
s^0	Minimum bumper-to-bumper gap for completely stopped traffic(m)