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Branch switching at Hopf bifurcation analysis via asymptotic numerical method: Application to nonlinear free vibrations of rotating beams

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ABSTRACT

This paper deals with the computation of backbone curves bifurcated from a Hopf bifurcation point in the framework of nonlinear free vibrations of a rotating flexible beams. The intrinsic and geometrical equations of motion for anisotropic beams subjected to large displacements are used and transformed with Galerkin and harmonic balance methods to one quadratic algebraic equation involving one parameter, the pulsation. The latter is treated with the asymptotic numerical method using Padé approximants. An algorithm, equivalent to the Lyapunov–Schmidt reduction is proposed, to compute the bifurcated branches accurately from a Hopf bifurcation point, with singularity of co-rank 2, related to a conservative and gyroscopic dynamical system steady state, toward a nonlinear periodic state. Numerical tests dealing with clamped, isotropic and composite, rotating beams show the reliability of the proposed method reinforced by accurate results.

1. Introduction

The study of vibrations of flexible rotating beams is of great importance in the design, optimization, and control of a variety of rotating structures such as turbine blades, robot arms, wind turbines, and helicopter rotor blades. This interest is evidenced by the number of papers devoted to the study of nonlinear aspects, the coupling effects of gyroscopic terms, and to the determination of their vibration characteristics, namely natural frequencies and mode shapes, which are crucial for modeling. Earlier studies treat the problem through a linear model [1–4]. Recent studies have focused on the nonlinear vibrations of rotating beams and most of them are based on different nonlinear dynamical models to construct nonlinear normal modes (NNMs). In [5], the authors used the Euler–Bernoulli beam model and asymptotic series to approximate the geometry and the invariant manifolds to construct the nonlinear normal modes. Ref. [6] shows that the Galerkin-based approach presents a more exact reduction than the asymptotic series used in [5], particularly for a large amplitude. In [7], the beam model was based on the Von Karman strain–displacement relationships. The authors applied the method of multiple scales (MMS) on the Galerkin-reduced discretized version of the equation of motion to construct the NNMs. Ref. [8] used dynamical model based on a geometrically exact approach and applied the MMS to construct the NNM.

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In rotating beams, as in most of the dynamic mechanical systems, gyroscopic effects are omnipresent, which lead to non-self-adjoint operators. This can be caused also by the application of the harmonic balance method on the differential governing equations to convert them into an algebraic equations. The resulting eigenvalues are complex conjugate pairs and consequently have complex conjugate eigenvectors both right and left [9]. When there is no damping, the eigenvalues are purely imaginary conjugate pairs. An example of such dynamic systems, is the intrinsic equations of nonlinear dynamics of anisotropic beams. These equations as presented in [10–12] contain gyroscopic terms and therefore present a nonlinear dynamic, gyroscopic, and conservative system.

The simple bifurcations (simple singular points) corresponding to co-rank 1 with one parameter and associated generally to free vibration and buckling of structures have been studied rather extensively using different continuation methods. For examples, there are studies dealing with nonlinear free vibration of beams and plates [13–15] using the arc-length continuation method, where the linear eigenvector corresponding to the considered eigenfrequency is used as first approximation of the starting point. In [16], the asymptotic numerical method is employed, based on the orthogonality conditions obtained from the Lyapunov–Schmidt reduction and with an initial displacement vector null, trivial solution, which is not always the case. In [17], the same techniques were used, as in [16], applied in fluid mechanics with some new reflections. These techniques are all based on the assumption that the kernel of the tangent operator is generated by one vector, which formally fails with a bifurcation problem with co-rank 2 or higher and must be improved.

A bifurcation problem with co-rank 2 is considered as a double eigenvalue problem or a double singular point problem in [18–20]. A particular case of this problem is considered in the numerical results Section 3, namely the nonlinear free vibration where the solution branch bifurcate from an equilibrium state point into a dynamic and periodic state. Hence, by definition it concerns a Hopf bifurcation problem [21,22], which is classified also as a dynamic bifurcation [23] and is characterized by a complex conjugate pair of purely imaginary eigenvalues for each mode. The Hopf bifurcation theorem is given in the literature, e.g. Theorem 2.11 in [22]. If one of the conditions is not respected, for example the transversality condition, the Hopf bifurcation point is called degenerate. It will be noted that when dealing with the Hopf bifurcations, there is equivalence between the time frequency domain and the time domain approaches [24].

In this paper, a new algorithm allowing the asymptotic numerical method to solve bifurcation problems with co-rank 2 with one parameter, which can be a milestone and extended to high-order problems with co-rank $n \geqslant 2$ is presented. As an application of this technique, the intrinsic formulation for dynamics of moving beams developed by Hodges [10,11] is applied to examples treated in the literature. This formulation is convenient for both isotropic and anisotropic beams with uniform cross-section subjected to large displacements and small strain without any geometrical approximations. It takes advantage of the one dimensional characteristic of a beam and does not require definitions of displacements and rotations; consequently, it is less expensive than using the three dimensional (3D) finite-element-method (FEM) analysis. This model has been used in nonlinear forced vibrations of rotating anisotropic beams study in [12] by coupling the Galerkin approach and the balance harmonic method and using two continuation methods: the pseudo-arclength and the asymptotic numerical method. The latter is mainly based upon the implicit function theorem, which holds for one dimensional kernel (co-rank 1) i.e. simple bifurcation, and is generalized for higher dimensional kernels (co-rank $\geqslant 2$), see Theorems I.5.1 and I.19.2 in [25]. The modal analysis studied in [12] concludes that the gyroscopic effects decrease the natural frequencies essentially for rotating Timoshenko beams.

The outline of this paper is as follows. In Section 2, the governing equations in the time domain are presented. Then, by using the Galerkin discretization approach and the harmonic balance method, the partial differential equations are reduced to one algebraic equation and therefore resolved in the frequency domain. For a nonlinear free vibration of a quadratic dynamical system, the Hopf bifurcation is identified. The asymptotic numerical method based upon the implicit function theorem and the Lyapunov–Schmidt reduction technique yields to post-bifurcated solution branches. To demonstrate the efficiency of our approach, numerical examples, treated in the literature, are presented in Section 3, which deals with isotropic and composite uniform cross-section beams. Finally, Section 4 is devoted to the conclusion.

2. Mathematical formulation

2.1. Governing equations

The nonlinear equations of motion without external forces derived from Hamilton's principle and the nonlinear kinematical equations are [10–12]:

$$F' + (\tilde{\Bbbk} + \tilde{\kappa})F = \dot{P} + \tilde{\Omega}P \tag{1}$$

$$M' + (\tilde{\Bbbk} + \tilde{\kappa})M + (\tilde{e_1} + \tilde{\gamma})F = \dot{H} + \tilde{\Omega}H + \widetilde{V}P \tag{2}$$

$$V' + (\tilde{\Bbbk} + \tilde{\kappa})V + (\tilde{e_1} + \tilde{\gamma})\Omega = \dot{\gamma} \tag{3}$$

$$\Omega' + (\tilde{\Bbbk} + \tilde{\kappa})\Omega = \dot{\kappa} \tag{4}$$

Using the Galerkin approach and the harmonic balance method (see Appendix A and [12]), this system of partial differential equations Eqs. (1)–(4) is reduced to one nonlinear algebraic equation in the frequency domain with one parameter ω,