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Eigenvalue based approach to bounded synchronization of asymmetrically coupled networks

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ABSTRACT

This paper investigates global synchronization of asymmetrically coupled dynamical networks with nonidentical nodes in the sense of boundedness. A novel bounded synchronization criterion is presented by checking the validity of inequality involving the second smallest eigenvalue of a redefined symmetric matrix associated with the asymmetric Laplacian matrix. In particular, this criterion can be used to determine the global exponential synchronization of asymmetrically coupled networks with identical nodes by the proposed symmetrization operation, without assuming the symmetry or irreducibility of the coupling matrix. Comparing with the existing contributions, our synchronization result is less conservative and can overcome the complexity of calculating eigenvalues of an asymmetric Laplacian matrix. Numerical experiments are carried out to demonstrate the effectiveness of the method.

1. Introduction

Over the past few years, the study of complex systems from the viewpoint of complex networks has become an important interdisciplinary issue. The approach is to simplify complex systems by complex networks with individual units called nodes, connected by links that exhibit complex topological properties [1,2]. Thereafter, synchronization in large-scale complex systems of dynamically interacting units has received a great deal of attention extensively in recent years [3–6], and insightful findings regarding the collective behavior on complex networks have been reported [7–15]. Typically, Pecora and Carroll proposed a master stability function approach to investigation of the local stability of synchronization manifold by calculating the transversal Lyapunov exponents [7]. This method has been widely used to explore local synchronization in symmetrically coupled or asymmetrically coupled networks [16–18]. Besides, some global synchronization results have also been developed for complex coupled networks, e.g. Refs. [19–23].

Generally speaking, synchronization of complex networks is determined by the dynamics of isolated nodes and the coupling topology between nodes. Most efforts have been put on the study of the coupling topology by assuming that all the node dynamics are identical, and thereby topology of a complex network is easier to be revealed as an interplay entity between the dynamical states of nodes and interaction patterns. However, it is not a typical case to assume that all the nodes are identical in biology, society and various engineering systems. Significant differences commonly exist within the relevant individual nodes. For instance, the genetic biological oscillators, even in the same species, are usually nonidentical possibly

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due to heterogeneous nutrition conditions and fluctuated environments [24]. In power systems, the generators and loads are connected to buses which are interconnected by transmission lines in a network structure [25]. Thus, the power systems can be viewed as a dynamical network with nonidentical nodes.

There is no doubt that the behavior of a complex heterogeneous network is much more complicated than that of the identical-node case. A simple case is that all nonidentical nodes share the common equilibrium, and the stability of dynamical networks with nonidentical nodes has been studied by Lyapunov method [25–27]. For general cases, the invariant synchronization manifold, which guarantees by the diffusive condition in complex networks with identical nodes, usually disappears for nonidentical-node case due to the heterogeneity of isolated nodes, and the ultimate synchronous trajectory, in general, has to be confined to some particular solution or resorted to the external controllers [28–33]. However, a complex heterogeneous network of coupled systems may still exhibit some kind of synchronous behaviors: diverse biochemical rhythms are generated by lots of cellular oscillators that somehow manage to operate synchronously [34,35]. Numerical simulations have been respectively examined for nonidentical genetic oscillators and Kuramoto oscillators etc [36–38]. Li et al. presented a theoretical method for exploring the synchronization of coupled nonidentical genetic oscillators that can be transformed into Lur'e form [39]. Zhao et al. addressed the problem of bounded synchronization for general dynamical networks with nonidentical nodes by introducing the average dynamics of all nodes [40]. But it fails to work for those networks with asymmetric coupling configurations since decompositions into a few of lower dimensional subsystems are no longer possible even for the local synchronization. This paper is an attempt to investigate bounded synchronization of asymmetrically coupled networks.

In this paper, without assuming symmetry or irreducibility of coupling configurations, a new eigenvalue based bounded synchronization criterion is deduced for the asymmetrically coupled network with nonidentical nodes by employing the Lyapunov function approach. Under some mild assumptions, the second smallest eigenvalue of a redefined symmetric matrix associated with the asymmetric Laplacian matrix is used to assess the converge domain of bounded synchronization. In particular, if the considered network consists of linearly coupled identical nodes, then the obtained result reduces to the synchronization criterion. Compared with existing results, the criterion exhibits less conservativeness, and can be analytically applied to any asymmetrically coupled networks by overcoming the complexity of calculating eigenvalues of the asymmetric Laplacian matrix.

The rest of this paper is organized as follows. Section 2 presents some mathematical preliminaries. The main theoretical results are presented in Section 3. Numerical validations are given to show the effectiveness of the presented results in Section 4. Section 5 concludes the investigation.

2. Preliminaries

Consider a dynamical network consisting of N linearly and diffusively coupled nonidentical nodes, with each node being an n-dimensional dynamical system, described by

$$\dot{x}_i(t) = f_i(t, x_i(t)) - c\sum_{j=1}^{N} l_{ij}\Gamma x_j(t), \quad i = 1, \ldots, N, \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^{\mathrm{T}} \in R^n$ denotes the state vector of the i-th node, $f_i(t, x_i(t)) : R^+ \times R^n \to R^n$ with $i = 1, \ldots, N$ is a smooth nonlinear vector-valued function describing the nodal self-dynamics, the constant $c > 0$ is the coupling strength of the whole network, the inner-coupling matrix $\Gamma = \mathrm{diag}(\gamma_1, \ldots, \gamma_n) \in R^{n\times n}$ describes the way of linking the components in each pair vector of nodes with $\gamma_i \geqslant 0$. $L = (l_{ij})_{N\times N}$ is the Laplacian matrix with zero-sum rows, representing the coupling configuration of network (1): If there is a directed connection from node j to node i, $l_{ij} < 0$ $(i \neq j)$; else $l_{ij} = 0$.

In order to facilitate the discussions after-mentioned, we decompose the asymmetric matrix L into two $N \times N$ matrices $\bar{L}$ and $\underline{L}$ [20]:

$$L = \bar{L} + \underline{L}, \tag{2}$$

where symmetric matrix $\bar{L} = (\bar{l}_{ij})_{N\times N}$ and matrix $\underline{L} = (\underline{l}_{ij}))_{N\times N}$ both satisfy the zero-row-sum condition with off-diagonal entries defined as $\forall i \neq j$,

$$\bar{l}_{ij} = \frac{1}{2}(l_{ij} + l_{ji}) = \bar{l}_{ji}, \tag{3}$$

$$\underline{l}_{ij} = \frac{1}{2}(l_{ij} - l_{ji}) = -\underline{l}_{ji}. \tag{4}$$

Throughout the rest of the paper, we have the following assumption.

Hypothesis 1. Suppose that network (1) is weakly connected, i.e., there always exists a path between any two nodes in network (1) if replacing all of its directed edges with undirected edges.

It is well-known that the Laplacian matrix L is irreducible if and only if network (1) is strongly connected, i.e., there is a path from each node to every other node. Note that in Hypothesis 1, we do not assume matrix L to be irreducible or symmetric. Under Hypothesis 1, the Laplacian L has a simple eigenvalue zero with $\xi = \frac{1}{\sqrt{N}}(1, 1, \ldots, 1)^{\mathrm{T}} \in R^N$ as the corresponding eigenvector, and the real parts of all the other eigenvalues are positive, i.e., the eigenvalue of L satisfy $0 = \lambda_1(L) < \mathcal{R}(\lambda_2(L)) \leqslant \cdots \leqslant \mathcal{R}(\lambda_N(L))$.