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An analytical method for analyzing symmetry-breaking bifurcation and period-doubling bifurcation

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ABSTRACT

A new modification of homotopy analysis method (HAM) is proposed in this paper. The auxiliary differential operator is specifically chosen so that more than one secular term must be eliminated. The proposed method can capture asymmetric and period-2 solutions with satisfactory accuracy and hence can be used to predict symmetry-breaking and period-doubling bifurcation points. The variation of accuracy is investigated when different number of frequencies are considered.

1. Introduction

Bifurcation refers to a qualitative change in periodic orbits (or equilibrium points), or in their stability attributes, caused by a small variation in system parameters. Bifurcations are important phenomena which exist in the behavior of many nonlinear systems and are closely related to system stability and more complicated behavior such as chaos. If an analytical method cannot be used to investigate bifurcations, then the understanding of nonlinear systems that it brings to us will be insufficient. Although there is a vast literature [1–6] investigating nonlinear systems through analytical methods, few studies attempted to investigate bifurcation phenomena with these analytical methods. Tesi et al. proposed to use a combination of the Loeb criterion, a criterion for numerically determining the stability of a limit cycle, and the first-order harmonic balance approximation to investigate period-doubling bifurcations and showed that this method could be applied in conjunction with a control design to delay or even eliminate a period-doubling cascade to chaos [7]. Bonani and Gilli put forward an approach for investigating limit-cycle bifurcations in nonlinear control systems, which not only detected bifurcations of the limit cycles, but also calculated the Floquet multipliers as the roots of an algebraic equation as well as determined the stability [8]. Berns et al. proposed a quasi-analytical computational scheme for predicting period-doubling bifurcation based one high-order harmonic balance analysis and characteristic multiplier tracking [9]. Luo and Huang analytically predicted period-m solutions of a periodically excited Düffing oscillator with the generalized harmonic balance method [10]; Luo and Huang presented the analytical expressions for period-m flows and chaos of a damped Düffing oscillator subjected to harmonic excitation [11]; Luo and Huang derived the analytical period-1 solution to a Düffing oscillator with harmonic excitation and a twin-well potential and studied the routes of the period-1 motion to chaos [12]; the three papers successfully derived analytical period-m solutions and predicted their stability and bifurcations as well the routes to chaos.

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Analytical study on the dynamics of nonlinear systems dates back to early celestial mechanics. In 1830, Poisson first introduced an early form of the perturbation methods which obtained periodic solutions based on a power series expansion with respect to a small change of a parameter in the nonlinear system. Lindstedt put forward a variant of the perturbation method without strict mathematical foundation [13]. It was not until 1892 that the mathematical foundation of the perturbation method was established by Poincaré [14]. Standard perturbation methods like the Lindstedt–Poincaré method work very well for celestial bodies where the oscillations are subjected to weak nonlinearity most of the time, but will yield large errors when the investigated system has strong nonlinearity. However, in many disciplines like mechanical engineering, earthquake engineering, and structural engineering, strongly nonlinear dynamic systems are very common. Therefore, numerous studies have attempted to find analytical methods for solving strongly nonlinear differential equations. Burton proposed a modified Lindstedt–Poincaré method which can be applied to systems with strong nonlinearity [15]. The standard method of multi-scale was modified by Burton and Rahman to enable accurate periodic solutions to be acquired for strongly nonlinear oscillators [16]. Cheung et al. introduced a new parameter which can always be kept small regardless of the magnitude of the original system parameters [17]. Liao combined the concept of homotopy in topology with Maclaurin series expansion and developed a method called the homotopy analysis method (HAM) [18]. HAM also works for strongly nonlinear systems [19–21]. Although a number of analytical methods for strongly nonlinear systems have been put forward, they can only be used to obtain periodic solutions of some selected nonlinear problems, which are mostly period-one solutions. Most of the bifurcation phenomena like symmetry-breaking bifurcations and period-doubling bifurcations are beyond the capability of these existing analytical methods.

Nayfeh and Balachandran described a route to chaos, which is now basically the most well-known one [22]. This route to chaos starts with a symmetry-breaking bifurcation and period-doubling bifurcations. Typically a symmetry-breaking bifurcation occurs prior to a period-doubling bifurcation, which is followed by a cascade of period-doubling bifurcations [23,24]. Finally it reaches the onset of chaos. One of the examples reported in [22] is for the following Düffing oscillator.

$$\ddot{x} + c\dot{x} + x - x^3 = F\cos\Omega t. \tag{1}$$

For the Düffing oscillator represented by Eq. (1), parameters held constant are c at 0.4 and Ω at 0.8. When F is equal to 0.350, one stable symmetric period-one oscillation occurs. Its limit cycle and amplitude of frequency components are presented in Fig. 1(a) and (b), respectively.

When F increases to 0.380, the steady-state solution becomes asymmetric as shown in Fig. 2(a), which suggests a symmetry-breaking bifurcation has already occurred. The appearance of a constant term can be easily seen as a zeroth order harmonic in Fig. 2(b).

When the excitation amplitude F further increases to 0.386, a period-2 steady-state solution arises as shown in Fig. 3(a), which suggests a period-doubling bifurcation occurs between $F = 0.380$ and $F = 0.386$. The appearance of a frequency component at $\frac{\Omega}{2}$ can be seen in Fig. 3(b).

In the present paper, a new analytical procedure for obtaining approximate solutions for strongly nonlinear problems will be outlined in the framework of HAM. This new procedure will be applied to investigate symmetry-breaking bifurcations and period-doubling bifurcations of a Düffing oscillator.

2. Formulation of multi-frequency HAM

To start the procedure of multi-frequency HAM (MFHAM), an auxiliary linear differential operator conveying information of n (n is an arbitrary positive integer) fundamental frequencies needs to be constructed.

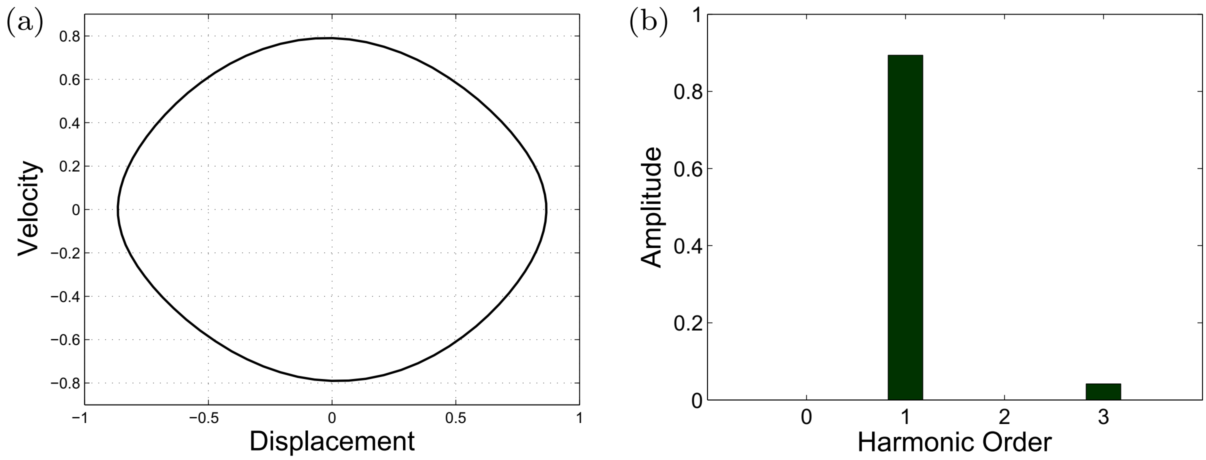

Fig. 1. The limit cycle of a symmetric period-one solution. (a) A symmetric limit cycle and (b) harmonic components.