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The control method for the lattice hydrodynamic model

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ABSTRACT

The delayed-feedback control method is applied for lattice hydrodynamic model of traffic flow. The linear stability condition with and without control signal are derived through linear and nonlinear analysis. Numerical simulation is carried out and the results confirm that the traffic congested can be suppressed efficiently by considering the control signal.

1. Introduction

Generally speaking, traffic problems have attracted considerable attention and the properties of traffic jams are important, which have been studied by some traffic models: the car-following models, the cellular automaton models, the lattice hydrodynamic models, and the hydrodynamic models [1–8]. The suppression of the traffic congestion is a vital issue for modern traffic.

The coupled-map (CM) car-following model [9,10] is a discrete-time version of the original optimal velocity (OV) model. In 1999, Konishi et al. put forward a nonchaotic CM car-following (KKH, for short) model [11] and analyzed the traffic jam phenomena for open flow theoretically and proposed a scheme for suppression of the traffic jam by applying a decentralized delayed-feedback control. In 2006, Zhao and Gao [12] proposed a simple control model for the suppression of the traffic congestion. The CM car-following model for traffic flow with the consideration of the application of ITS were proposed by Ge and Han [13,14]. In 2012, Ge [15] presented a simple control method to suppress traffic jam for car-following model. CM car-following model and car following model belong to microscopic models. However, no studies have ever tried to analyze the traffic jam for macroscopic model by using the delayed-feedback control theory.

The lattice model is firstly proposed by Nagatani [16,17] to analyze the jamming transition evolution of traffic flow, which belongs to macroscopic model. Afterwards, a lot of extended lattice models [18–24] have been developed by taking different factors into account. The lattice hydrodynamic model has many similar properties with car following model, such as the linear stability, the density waves et al. In view of this, we want to investigate the mechanism of the traffic jam by use of the delayed-feedback control theory based on the lattice hydrodynamic model.

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The outline of this paper is as follows. In Section 2, The classical lattice hydrodynamic model is recovered. In Section 3, the linear stability of the model is analyzed by using control theory. In Section 4, the control signal is added to the model and the linear stability of the new model is studied again. Numerical simulations are carried out for the lattice hydrodynamic model with and without control signal in Section 5. The conclusion are given in Section 6.

2. Model

The basic model is the simplified version of the hydrodynamic model proposed by Nagatani [16,17]. The governing equations are described as follows:

$$\partial_t \rho_j + \rho_0(\rho_j v_j - \rho_{j-1} v_{j-1}) = 0, \tag{1}$$

$$\partial_t(\rho_j v_j) = a\rho_0 V(\rho_{j+1}) - a\rho_j v_j, \tag{2}$$

where $a = 1/\tau$ is the sensitivity of a driver, ρ_0 is the average density, ρ_{j+1} is the local density at position $j+1$ at time t, and local density ρ_{j+1} is related with the inverse of headway $h(x,t)$. The right side of Eq. (2) expresses the tendency of traffic flow ρv at a given density to relax to some natural average flow $\rho_0 V(\rho_{j+1})$. We rewrite the Eqs. (1) and (2) as the traffic flow version, which is

$$\partial_t \rho_{j+1} + \rho_0(q_{j+1} - q_j) = 0, \tag{3}$$

$$\partial_t(q_j) = a\rho_0 V(\rho_{j+1}) - aq_j, \tag{4}$$

we adopt the similar optimal velocity function as that used by Bando et al. [8]:

$$V(\rho) = (v_{max}/2)[tanh(1/\rho - 1/\rho_c) + tanh(1/\rho_c)], \tag{5}$$

where ρ_c is the safety density. Assuming that the desired density and flux of the traffic flow system has the steady-state uniform flow solution in the following form

$$[\rho_n, q_n]^T = [\rho_n^*, q_n^*]^T. \tag{6}$$

3. Linear stability analysis

The control method in our model is similar to the rule of velocity difference in the car-following model with control method. We apply the linear stability method with control theory to the traffic flow model described by Eqs. (4) and (5). Let Eqs. (4) and (5) be linearized around steady state and the linearized vehicular dynamics can be rewritten as follows:

$$\partial_t \rho_{j+1}^0 + \rho_0(q_{j+1}^0 - q_j^0) = 0, \tag{7}$$

$$\partial_t(q_j^0) = a\rho_0 \Lambda_{n+1} \rho_{j+1}^0 - aq_j^0, \tag{8}$$

where $\Lambda_{n+1} = \frac{\partial V(\rho_{n+1})}{\partial \rho_{n+1}}\Big|_{\rho_{n+1}=\rho^*}, \rho_{j+1}^0 = \rho_{j+1} - \rho^*, q_j^0 = q_j - q^*, q_{j+1}^0 = q_{j+1} - q^*$.

Taking Laplace transform as L, we have

$$sP_{j+1}(s) + \rho_{j+1}(0) + \rho_0(Q_{j+1}(s) - Q_j(s)) = 0, \tag{9}$$

$$sQ_j(s) + q_j(0) = a\rho_0 \Lambda_{n+1} P_{j+1}(s) - aQ_j(s), \tag{10}$$

where $L(\rho_{j+1}) = P_{j+1}(s)$, $L(q_{j+1}) = Q_{j+1}(s)$, $L(q_j) = Q_j(s)$, L denotes the Laplace transform. By eliminating density in Eqs. (9) and (10), one obtains the flux equation:

$$Q_j(s) = \frac{-a\rho_0^2\Lambda}{s^2 + as - a\rho_0^2\Lambda} Q_{j+1}(s) + \frac{a\rho_0}{s^2 + as - a\rho_0^2\Lambda}\rho_{j+1}(0) + \frac{s}{s^2 + as - a\rho_0^2\Lambda} q_j(0), \tag{11}$$

where transfer function $G(s)$ is $\frac{-a\rho_0^2\Lambda}{s^2+as-a\rho_0^2\Lambda}$ and the characteristic polynomial $d(s) = s^2 + as - a\rho^2\Lambda$. From Definition 1 in Ref. [11], it is known that if the characteristic polynomial $d(s)$ is stable and the H_∞ norm of transfer function $G(s)$ is equal to or less than 1, then traffic jams will be weaken. So we can get the stability condition

$$a > -2\rho_0^2\Lambda.$$

If the condition $a > -2\rho_0^2\Lambda$ is satisfied, we can obtain $\left\|\frac{a\rho_0}{s^2+as-a\rho_0^2\Lambda}\right\|_\infty < 1$ and $\left\|\frac{s}{s^2+as-a\rho_0^2\Lambda}\right\|_\infty < 1$, that is to say, the flux disturbance due to $\rho_{j+1}(0)$ and $q_j(0)$ would not be amplified when it propagates backwards.

Upon the aforementioned linear analysis, the stability condition can be abbreviated as following: