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Simulating nonlinear aeroelastic responses of an airfoil with freeplay based on precise integration method

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ABSTRACT

This paper proposed a numerical algorithm based on precise integration method to investigate the aeroelastic system of an airfoil with a freeplay. The system was split into three linear sub-systems separated by switching points related with the freeplay. A predictor–corrector algorithm was constructed to tackle the key computational obstacle in accurately searching system responses passing the switching points. With the aid of the algorithm, the precise integration method can solve the sub-systems one by one and provide solutions to any desired accuracy compared with exact solutions. Moreover, it can keep high precision with the step length increasing. The precise integration method is more accurate and efficient than the Runge–Kutta method with the same time step. In addition, the Runge–Kutta sometimes provides limit cycle oscillations, bifurcation charts or chaotic responses falsely even though the step length is much smaller than that adopted in precise integration method. Due to the high precision and efficiency, the presented approach has potential to become a benchmark for solution techniques for piecewise nonlinear dynamical systems.

1. Introduction

Predicting the nonlinear aeroelastic responses of an airfoil has stimulated the curiosity and interests of many researchers for years [1,2]. Liu and Dowell [3] investigated the aeroelastic system with a cubic pitching stiffness by the harmonic balance method. The harmonic balancing was combined with an optimization technique by Chen et al. [4] for solving an airfoil-store aeroelastic witch cubic stiffness. Equivalent linearization [5] can also be applied in solving nonlinear aeroelastic systems. A detailed bibliography on this topic was given in Refs. [1,2].

In most cases, it is cumbersome and expensive, or even impossible, to obtain exact solutions of nonlinear aeroelastic systems. It is therefore necessary to employ numerical techniques in validating the results provided by the analytical or semi-analytical approaches [1–5]. Numerical simulations in nonlinear aeroelastics can be performed based on the state space models [6,7]. These models were generally solved by time-marching integration techniques such as the Runge–Kutta (RK) and Newmark methods, etc [8–11].

The precise integration method (PIM) initiated by Zhong [12] has been widely applied to various problems modeled by ordinary differential equations such as structural dynamics, optimal control, and flexible multi-body dynamics problems, etc [13–17]. This method is famous for its high accuracy and computation efficiency. In this paper, the PIM was applied to

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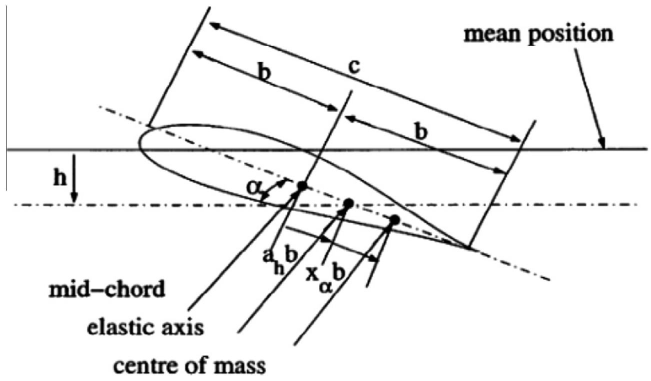

Fig. 1. The sketch of an airfoil oscillating in pitch (α) direction with respect to the elastic axis, and in plunge (h) direction measured from the mean position.

simulate the aeroelastic responses of an airfoil with a freeplay. This study was motivated by the locally linear property of the aeroelastic system with a piecewise freeplay and the fact that the PIM was extremely suitable for linear systems.

The aeroelastic system was first divided into three subsystems according to the freeplay. Between the subsystems, there are switching points. A troublesome problem in simulating piecewise linear systems was confronted in determining the vibration response passing the switching points [18,19]. Lin and Cheng [18] found that an entirely incorrect asymptotic behavior can occur due to the accumulative error in tracking switching points by the RK method. Significant discrepancies between the exact and numerical solutions may sometimes be observed. In order to tackle this problem, a predictor–corrector algorithm was proposed in Section 3.

2. Equations of motions

We considered the two-degree-of-freedom airfoil oscillating in pitch and plunge directions. The symbols appear in this model are given in Fig. 1.

The pitch angle about the elastic axis is denoted by α, positive with the nose up; the plunge deflection is denoted by h, positive in the downward direction. The elastic axis is located at a distance $a_h b$ from the mid-chord, and the mass center is located at a distance $x_\alpha b$ from the elastic axis. In terms of non-dimensional time $t = \omega_\alpha t_1$ (t_1 is the real time) and non-dimensional plunge displacement $\xi = h/b$, the coupled motions of the airfoil in incompressible unsteady flow can be modeled as follows

$$\begin{aligned} &c_0\ddot{\xi} + c_1\ddot{\alpha} + \left(c_2 + 2\zeta_\xi \frac{\bar{\boldsymbol{\omega}}}{\boldsymbol{U}^*}\right)\dot{\xi} + c_3\dot{\alpha} + c_4\xi + c_5\alpha + c_6 w_1 + c_7 w_2 + c_8 w_3 + c_9 w_4 + \left(\frac{\bar{\boldsymbol{\omega}}}{\boldsymbol{U}^*}\right)^2 G(\xi) = f(t) \\ &d_0\ddot{\xi} + d_1\ddot{\alpha} + d_2\dot{\xi} + \left(d_3 + 2\zeta_\alpha \frac{1}{\boldsymbol{U}^*}\right)\dot{\alpha} + d_4\xi + d_5\alpha + d_6 w_1 + d_7 w_2 + d_8 w_3 + d_9 w_4 + \left(\frac{1}{\boldsymbol{U}^*}\right)^2 M(\alpha) = g(t) \end{aligned} \tag{1}$$

where the superscript denotes the differentiation with respect to t; $\boldsymbol{U}^*$ is a non-dimensional velocity defined as $U^* = U/b\omega_\alpha$, and $\bar{\omega}$ is given by $\bar{\omega} = \omega_\xi/\omega_\alpha$. Symbols ω_ξ and ω_α are the natural frequencies of the uncoupled plunging and pitching modes respectively; ζ_α and ζ_ξ are the damping ratios; $G(\xi)$ and $M(\alpha)$ represent the nonlinear plunge and pitch stiffness terms, respectively. The coefficients $c_0 \sim c_9, d_0 \sim d_9$ can be referred to Liu and Dowell [3]. Parameters w_i's that depend upon ξ and α are given as

$$\begin{aligned} w_1 &= \int_0^t e^{-\varepsilon_1(t-\sigma)}\alpha(\sigma)d\sigma, \quad w_2 = \int_0^t e^{-\varepsilon_2(t-\sigma)}\alpha(\sigma)d\sigma \\ w_3 &= \int_0^t e^{-\varepsilon_1(t-\sigma)}\xi(\sigma)d\sigma, \quad w_4 = \int_0^t e^{-\varepsilon_2(t-\sigma)}\xi(\sigma)d\sigma \end{aligned} \tag{2}$$

with the constants as $\varepsilon_1 = 0.0455$, $\varepsilon_2 = 0.3$. By introducing a variable vector $X = (x_1, x_2, \ldots, x_8)^T$ with $x_1 = \alpha, x_2 = \dot{\alpha}, x_3 = \xi, x_4 = \dot{\xi}, x_5 = w_1, x_6 = w_2, x_7 = w_3, x_8 = w_4$, the coupled state space system given by Eq. (1) can be rewritten as a set of eight first-order ordinary differential equations as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = a_{21}x_1 + (a_{22} - 2jc_0\zeta_\alpha\frac{1}{U^*})x_2 + a_{23}x_3 + (a_{24} + 2jd_0\zeta_\xi\frac{\bar{\omega}}{U^*})x_4 + a_{25}x_5 + a_{26}x_6 + a_{27}x_7 + a_{28}x_8 + j\left(d_0\left(\frac{\bar{\omega}}{U^*}\right)^2 x_3 - c_0\left(\frac{1}{U^*}\right)^2 M(x_1)\right) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = a_{41}x_1 + (a_{42} + 2jc_1\zeta_\alpha\frac{1}{U^*})x_2 + a_{43}x_3 + (a_{44} - 2jd_1\zeta_\xi\frac{\bar{\omega}}{U^*})x_4 + a_{45}x_5 + a_{46}x_6 + a_{47}x_7 + a_{48}x_8 + j\left(c_1\left(\frac{1}{U^*}\right)^2 M(x_1) - d_1\left(\frac{\bar{\omega}}{U^*}\right)^2 x_3\right) \\ \dot{x}_5 = x_1 - \varepsilon_1 x_5 \\ \dot{x}_6 = x_1 - \varepsilon_2 x_6 \\ \dot{x}_7 = x_3 - \varepsilon_1 x_7 \\ \dot{x}_8 = x_3 - \varepsilon_2 x_8 \end{cases} \tag{3}$$