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Local weak form meshless techniques based on the radial point interpolation (RPI) method and local boundary integral equation (LBIE) method to evaluate European and American options

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ABSTRACT

For the first time in mathematical finance field, we propose the local weak form meshless methods for option pricing; especially in this paper we select and analysis two schemes of them named local boundary integral equation method (LBIE) based on moving least squares approximation (MLS) and local radial point interpolation (LRPI) based on Wu's compactly supported radial basis functions (WCS-RBFs). LBIE and LRPI schemes are the truly meshless methods, because, a traditional non-overlapping, continuous mesh is not required, either for the construction of the shape functions, or for the integration of the local sub-domains. In this work, the American option which is a free boundary problem, is reduced to a problem with fixed boundary using a Richardson extrapolation technique. Then the θ-weighted scheme is employed for the time derivative. Stability analysis of the methods is analyzed and performed by the matrix method. In fact, based on an analysis carried out in the present paper, the methods are unconditionally stable for implicit Euler ($\theta = 0$) and Crank–Nicolson ($\theta = 0.5$) schemes. It should be noted that LBIE and LRPI schemes lead to banded and sparse system matrices. Therefore, we use a powerful iterative algorithm named the Bi-conjugate gradient stabilized method (BCGSTAB) to get rid of this system. Numerical experiments are presented showing that the LBIE and LRPI approaches are extremely accurate and fast.

1. Introduction

Over the last thirty years, financial derivatives have raised increasing popularity in the markets. In particular, large volumes of options are traded everyday all over the world and it is therefore of great importance to give a correct valuation of these instruments.

Options are contracts that give to the holder the right to buy (call) or to sell (put) an asset (underlying) at a previously agreed price (strike price) on or before a given expiration date (maturity). The majority of options can be grouped in two categories: European options, which can be exercised only at maturity, and American options, which can be exercised not only at maturity but also at any time prior to maturity.

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Options are priced using mathematical models that are often challenging to solve. In particular, the famous Black–Scholes model [1] yields explicit pricing formulae for some kinds of European options, including vanilla call and put, but the modeling of American options is quite complicated. Hence, an analytical solution is impossible. Therefore, to solve this problem, we need to have a powerful computational method. To this aim, the most common approaches are the finite difference/finite element/finite volume methods/fast Fourier transform (see, e.g., [2–19]) and the binomial/triniomial trees (see, e.g. [20–23]), nevertheless some authors have also proposed the use of meshless algorithms based on radial basis functions [24–29] and on quasi radial basis functions [30].

Recently, a great attention has been paid to the development of various meshless formulations for solution of boundary value problems in many branches of science and engineering. Meshless methods are becoming viable alternatives to either finite element method (FEM) and boundary element method (BEM). Compared to the FEM and the BEM, the key feature of this kind of method is the absence of an explicit mesh, and the approximate solutions are constructed entirely based on a group of scattered nodes. Meshless methods have been found to possess special advantages on problems to that the conventional mesh-based methods are difficult to be applied. These generally include problems with complicated boundary, moving boundary and so on [31–37]. A lot of meshless methods are based on a weak-form formulation on global domain or a set of local sub-domains.

In the global formulation background cells are needed for the integration of the weak form. Strictly speaking, these meshless methods are not truly meshless methods. It must be realized that integration is completed only those background cells with a nonzero shape function.

In the financial literature global meshless method (or Kansa method) has been proposed for pricing options under various models such as the Black–Scholes model, stochastic volatility models and Merton models with jumps. In particular, in the case of Black–Scholes model, we mention two papers by Hon and his co-author [30,26] where the global RBFs and quasi-radial basis functions are developed. Moreover, as regards the cases of stochastic volatility models and BlackScholes model on two underlying assets, new global meshless method is presented in Ballestra and Pacelli [25]. The techniques presented in [25] is combined of the Gaussian radial basis functions with a suitable operator splitting scheme. Also a numerical method has recently been presented by Saib et al. [38]. In particular, in this latter work, a differential quadrature radial basis functions is used to reduce the American option pricing problem under Merton's jump-diffusion model to a system of ordinary differential equations. The interested reader can also see [39,40].

In methods based on local weak-form formulation no cells are needed and therefore they are often known as truly meshless methods. By using a simple form for the geometry of the sub-domains, one can use a numerical integration method, easily. Recently, two family of meshless methods, on the basis of the local weak form for arbitrary partial differential equations with moving least-square (MLS) and radial basis functions (RBFs) approximation have been developed [41–45]. Local boundary integral equation method (LBIE) with moving least squares approximation and local radial point interpolations (LRPI) with radial basis functions have been developed by Zhu et al. [46] and Liu et al. [47,48], respectively. Both methods (LBIE and LRPI) are meshless, as no domain/boundary traditional non-overlapping meshes are required in these two approaches. Particularly, the LRPI meshless method reduces the problem dimension by one, has shape functions with delta function properties, and expresses the derivatives of shape functions explicitly and readily. Thus it allows one to easily impose essential boundary and initial (or final) conditions. Though the LBIE method is an efficient meshless method, it is difficult to enforce the essential boundary conditions for that the shape function constructed by the moving least-squares (MLS) approximation lacks the delta function property. Some special techniques have to be used to overcome the problem, for example, the Lagrange multiplier method and the penalty method [49]. In this paper, meshless collocation method is applied to the nodes on the boundaries. The papers of Zhu et al. [46,50,51] in linear and non-linear acoustic and potential problems, and for heat conduction problems, the works of Sladek brothers [52,53] by meshless LBIE are useful for researchers. This method has now been successfully extended to a wide rang of problems in engineering. For some examples of these problems, see [54–56] and other references therein. The interested reader of meshless methods can also see [57,58].

The objective of this paper is to extend the LRPI based on Wu's compactly supported radial basis functions (WCS-RBFs) with C^4 smoothness [59] and LBIE method based on moving least squares with cubic spline weight function to evaluate European and American options. To the best of our knowledge, the local weak form of meshless method has not yet been used in mathematical finance. Therefore, it appears to be interesting to extend such a numerical technique also to option valuation, which is done in the present manuscript. In particular, we develop a local weak form meshless algorithm for pricing both European and American options under the Black–Scholes model.

In addition, in this paper the infinite space domain is truncated to $[0, S_{max}]$ with a sufficiently large S_{max} to avoid an unacceptably large truncation error. The options' payoffs considered in this paper are non-smooth functions, in particular their derivatives are discontinuous at the strike price. Therefore, to reduce as much as possible the losses of accuracy the points of the trial functions are concentrated in a spatial region close to the strike prices. So, we employ the change of variables proposed by Clarke and Parrott [60].

As far as the time discretization is concerned, we use the θ-weighted scheme. Stability analysis of the method is analyzed and performed by the matrix method in the present paper. Furthermore, in this paper we will see that the time semi-discretization is unconditionally stable for implicit Euler ($\theta = 0$) and Crank–Nicolson ($\theta = 0.5$) schemes.

Finally, in order to solve the free boundary problem that arises in the case of American options is computed by Richardson extrapolation of the price of Bermudan option. In essence the Richardson extrapolation reduces the free boundary problem and linear complementarity problem to a fixed boundary problem which is much simpler to solve.

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