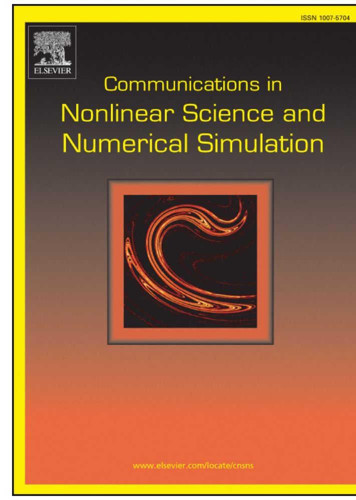

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Symmetry Analysis for a Class of Nonlinear Dispersive Equations

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A class of dispersive equations is studied within the framework of group analysis of differential equations. The enhanced Lie group classification is achieved. The complete list of equivalence transformations is presented. It is shown that certain equations from the class admit nonclassical reductions. Potential and potential nonclassical symmetries are also considered.

Keywords: Dispersive equations; Equivalence transformations; Lie symmetries; nonclassical reductions; Potential symmetries

1 Introduction

We consider the class of nonlinear dispersive equation [27]

$$u_t + \epsilon \left(u^m\right)_x + \frac{1}{b}\left[u^a\left(u^b\right)_{xx}\right]_x = 0 \tag{1}$$

which is of interest in mathematical physics. Special cases of this class have been used to model successfully physical situations in a wide range of fields. For example, if $a = 0,\ b = n$ we have the generalization of the KdV equation [22, 23]

$$u_t + \epsilon \left(u^m\right)_x + \frac{1}{n}\left(u^n\right)_{xxx} = 0 \tag{2}$$

and the equation that corresponds to the values $m = 2,\ a = b = 1$ describes a motion of a diluted suspension [28]. Equations of the type (2) with values of the parameters m and n are denoted by $K(m, n)$. For example, the properties of equation $K(2, 2)$ were examined in [22]. Further applications of the class (1) can be found in [25–27] and references therein.

Lie symmetries have been classified for many partial differential equations (PDEs) and for many classes of PDEs involving functions with a range of forms. Typically, extra symmetries exist for particular forms of these functions. The classical method of finding Lie symmetries is first to find infinitesimal transformations, with the benefit of linearization, and then to extend these to groups of finite transformations. This method is easy to apply and well established in the last decades [3–5, 12, 17, 18]. There is a continuing interest in finding exact solutions to nonlinear equations using Lie symmetries.

The goal of the present paper is to extend certain results of the recent work [6]. In particular, we present an enhanced Lie group classification for the class (1). In general, problems of group classification, except for trivial cases, are very difficult. This can be confirmed by a multitude of papers where classification problems are solved incompletely. For a modern treatment of group classification one can refer to [13, 29, 30], where generalised diffusion equations were considered. We construct the equivalence group using the direct method [14, 19, 29, 30] and the symmetry classification is carried out with respect to this group. We present the complete set of form-preserving (admissible) transformations. Furthermore the concept of nonclassical reductions is considered. Finally, the idea of potential symmetries, which is a class of nonlocal symmetries, is exploited for equations of the form (1) [2].