Benchmark solutions

Reprint of: A practical algorithm for real-time active sound control with preservation of interior sound

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ABSTRACT

In the active sound control problem a bounded domain is protected from noise generated outside via implementation of secondary sound sources on the perimeter. In the current paper we consider a quite general formulation in which sound sources are allowed to exist in the region to be shielded. The sound generated by the interior sources is considered as desired. It is required to remain it unaffected by the control in the protected area. This task proves to be much more complicated than the standard problem of active sound control because of the reverse effect of the controls on the input data. A novel practical algorithm is proposed that can be used for a real-time control. It accepts a preliminary tuning of the control system. In the algorithm the only input information eventually needed is the total acoustic field near the perimeter of the region to be shielded. It includes the contribution from both primary and secondary sources. In the algorithm the noise component to be attenuated is automatically extracted from the total acoustic field. The control system can potentially operate in a real-time regime since it only requires a consequent solution of a quadratic programming problem.

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1. Introduction

The active sound control (ASC) proved to be a very efficient approach to attenuate a low frequency noise. This approach has been intensively developed for the last fifty years. It provides an acoustic protection of a region from noise generated outside. The noise attenuation is realized via implementation of additional (secondary) sources on the perimeter of the region to be shielded. The approach is based on the use of Huygens' principle. As a result, secondary sources can be implemented along the perimeter to generate anti-noise. The required input information on the incoming noise can be immediately gained from the measurements. In case no desired acoustic field is presumed in the protected area, there are two principal approaches to the ASC: feed-back and feed-forward controls (see, e.g. [8,9]). The operation of the feed-back system is based on the minimization of the sound intensity at a set of sensors situated in the protected region. A common approach uses the filtered-x least mean square (FxLMS) algorithm [9] to minimize the level of noise. The feed-forward system operates in a predetermined way that depends on the acoustic field measured before the controls [10]. Practically the ASC is applica-

ble to the attenuation of low-frequency noise because of space and time limitations on the control system. Thus, the ASC can be an effective supplement to the passive control that is basically efficient in application to mid- and high-frequency noise (basically, above 500 Hz). Currently most applications are related to noise attenuation in ventilation systems, headphones and propeller driven aircrafts. It is to be noted most of modern ASC systems are adaptive to small changes in the system to be controlled [9].

The problem of ASC becomes much more complicated if interior sound sources are present. In particular, the feed-back approach becomes unapplicable since it is impossible to immediately distinguish the noise component to be minimized from the total field. There is an additional challenge if it is required to retain the interior (desired) sound unaffected by the controls. Apparently, for the first time this problem was formulated by Fedoryuk in [2]. Afterwards it has been considered in a number of publications for time-harmonic waves (see, e.g., [1,11,12,18]). The problem formulation was extended to composite regions in [13,14,20]. In [19] the nonlinear problem of ASC was tackled for the first time with the use of nonlinear potentials. The unsteady ASC problem was considered in [3,15–17]. The potential-based approach to ASC developed in these papers was realized in experiments [6,21–23]. In all these papers a local control is used. This means each control source can operate independently from the others on the basis of the local field from all primary desired and undesired sound sources. In fact

this is a strong assumption since the field generated by the primary sources cannot be immediately measured. As can be shown, if the interior sound exists, the local control inevitably must affect the input data [16]. Such a problem arises with noise attenuation in the cabin of a car or aircraft as well as with noise reduction in a room having an open window. Technically, this problem can be partially overcome with the use of directional measurements. This approach has been experimentally realized with a different level of success in [24–27]. A principally different approach is suggested in [28,29]. It requires a local solution of the wave equation across the boundary of the protected region with the controls. The boundary conditions for the wave equation are supposed to be taken from the measurements. The ASC problem is entirely formulated and considered in a discrete space.

In [5,7], nonlocal controls were derived in the frequency and time domains, respectively. In the approach the input data are supposed to be measured on the external side of the surface with the controls. The contribution of the desired field and controls is removed from the total field via the calculation of surface potentials which have a projection property. Thus, the control becomes nonlocal since it is based on the total field over the entire closed surface. This field is used as the density for the surface potentials. In the current paper we propose a practical algorithm to realize the control proposed in [5,7]. It is shown that to calculate the surface potentials the knowledge of Green's function is not necessary. Instead, a preliminary tuning of the control system can be used.

The rest of the paper is organized as follows. In the next section a mathematical formulation of the problem is given. The solution of the problem is based on the use of surface potentials, and in Section 3 a brief introduction to the generalized Caderón potentials is provided. The key property of the potentials is their projection property that is used for the ASC as described in Section 4. It is shown that the projection-based control is capable of retaining the interior sound field unaffected. Moreover, the approach can be applicable if only the total field from both primary and secondary sources is available nearby the perimeter of the region to be protected. A practical approach presumes the use of discrete sets of both sensors and controls. Such an algorithm is first described in Section 5 in the frequency domain. Then, in the next section, it is extended to an unsteady formulation.

2. Problem formulation

We suppose that some bounded region is acoustically protected from the noise generated outside. This means ideally there is no noise inside the protected region as the ASC operates. In addition, we allow a desired sound to be generated inside the protected region. It is required that the desired sound retains inside the protected region without changes. The boundary of the protected region is supposed to be acoustically transparent. The only way to tackle this problem is to distribute secondary acoustic sources outside the protected region. Practically the secondary sources should be situated on the boundary of the domain to be shielded in such a way that the total field satisfies the requirements formulated above.

Next, consider a mathematical formulation of the problem. For this purpose, introduce domain D: $\overline{D} \subseteq \mathbb{R}^3$ and a bounded subdomain $D^+ : \overline{D^+} \subset D$. We suppose that the boundaries of domains D and D^+ are the Lipschitz, and they are noted by Γ_0 and Γ, respectively.

Assume that sound field U is described by the following boundary-value problem (BVP) with homogeneous boundary conditions:
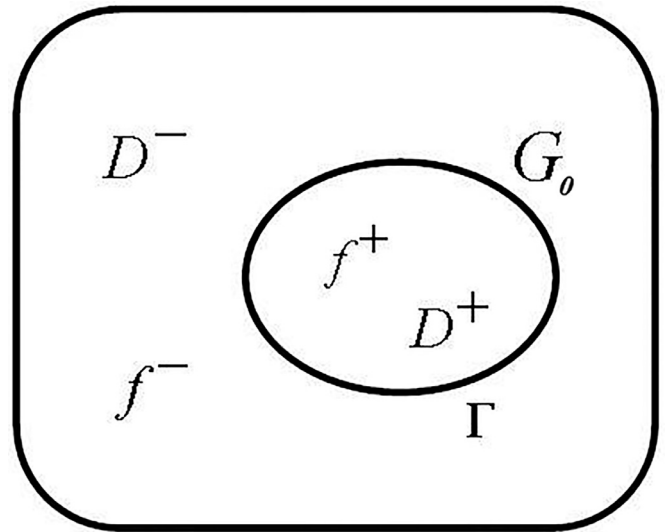
$$LU = f, \tag{1}$$

Fig. 1. Domain sketch.

$$U \in \Xi_D. \tag{2}$$

Here, the operator L is a linear differential operator. It can correspond to the linearized Euler equations (LEE) or the Helmholtz equation. Ξ_D is some functional linear space such that the inclusion (2) implicitly implies the boundary and initial (if needed) conditions.

To consider the LEE, introduce a first-order operator by

$$L_f \overset{def}{=} A^0 \frac{\partial}{\partial t} + \sum_1^3 A^i \frac{\partial}{\partial y^i}, \tag{3}$$

where $\{y^i\}$ $(i = 1, 2, 3)$ is the Cartesian coordinate system; A^k, $(k = 0, \ldots, 3)$ are 4×4 matrices: $A^k = A^k(\mathbf{y}) \in C^1(\overline{D})$. In the case of the unsteady formulation, we presume homogeneous initial conditions for the case of simplicity.

We also consider a second-order operator to analyze the Helmholtz equation:

$$L_s \overset{def}{=} -\nabla(p\nabla) - q, \tag{4}$$

where $p \in C^1(\overline{D})$, $q \in C(\overline{D})$ and $p > 0$.

In the further analysis, the solution of BVP (1) and (2) is considered in a weak sense. Thus, a function U is a solution of BVP (1) and (2) if $\langle LU, \Phi \rangle = \langle f, \Phi \rangle$ for any test function $\Phi(\overline{D}^0) \in C_0^\infty(\overline{D}^0)$. Here, $\langle f, \Phi \rangle$ denotes a linear continuous functional associated with a given generalized function f.

It is supposed that the acoustic sources are distributed both in D^+ and outside D^+ (see Fig. 1):

$$f = f^+ + f^-,$$
$$\text{supp} f^+ \subset D^+,$$
$$\text{supp} f^- \subset D^- \overset{def}{=} D \backslash \overline{D^+}. \tag{5}$$

Presume that we are going to protect region D^+ from noise generated outside D^+, in D^-. Thus, f^+ are interpreted as desired sources, while f^- generating noise.

The ASC problem can be formulated as an inverse source problem. It is required to find controls G_0 such as $\text{supp} G_0 \subset \Gamma$ and the solution of BVP:

$$LV = f + G_0,$$
$$V \in \Xi_D \tag{6}$$