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Electro-vortex flow simulation using coupled meshes

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ABSTRACT

A numerical model for simulating electro-vortical flows in OpenFOAM is developed. Electric potential and current are solved in coupled solid-liquid conductors by a parent-child mesh technique. The magnetic field is computed using a combination of Biot–Savart's law and induction equation. Further, a PCG solver with special regularisation for the electric potential is derived and implemented. Finally, a performance analysis is presented and the solver is validated against several test cases.

1. Introduction

Electro-vortex flow is highly relevant in many industrial processes. Possible applications span from electromagnetic stirring [1] for grain size reduction in solidification [2,3] over electrode welding [4], electroslag welding, electroslag (re-)melting [5,6], vacuum arc melting [7] to electrolytic reduction (of e.g. aluminium [8]). Further, many technical devices, as liquid fuses [9], electric jet engines, arc furnaces [10] and liquid metal batteries [11–13] involve or rely on electro-vortex flows. For an overview about such flows, see [14–16].

Electro-vortex flow is not an instability. It develops at (or near) a changing cross-section of a (liquid) conductor. Radial currents produce, together with their own magnetic field, a Lorentz force, which is non-conservative, i.e. its curl is not equal to zero. This force cannot be compensated totally by a pressure gradient and therefore drives a flow. For an illustrative example, see Shercliff [17].

Numerical simulation of electro-vortex flow is easy when modelling only the fluid, or a non-conducting obstacle inside a fluid. However, in most realistic cases, electric current passes from solid to liquid conductors and vice versa. The electric potential in these regions must therefore be solved in a coupled way. The classical, segregated approach means solving an equation in each region, and coupling the potential only at the interfaces by suitable boundary conditions [11]. While that is easy to implement, convergence is rather poor. An implicit coupling of the different regions by block matrices is a sophisticated alternative for increasing convergence [18]. However, it is memory-intensive and by no means easy to implement.

In this article we will present an alternative effective option for region coupling in OpenFOAM. We solve global variables (electric potential, current density) on a global mesh with a variable electric conductivity according to the underlying material. We then map the current density to the fluid regions and compute the electromagnetic induced flow there. This parent-child mesh technique was already used for the similar problem of thermal conduction [19,20] and just recently for the solution of eddy-current problems with the finite volume method [21].

2. Mathematical and numerical model

2.1. Overview

The presented multi-region approach is based on a single phase incompressible magnetohydrodynamic (MHD) model [11,22]. The flow in the fluid is described by the Navier–Stokes equation (NSE)

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla p + \nu\Delta\boldsymbol{u} + \frac{\boldsymbol{J}\times\boldsymbol{B}}{\rho}, \tag{1}$$

with $\boldsymbol{u}$ denoting the velocity, t the time, p the modified pressure, ν the kinematic viscosity and ρ the density. The fluid flow is modelled as laminar only; adding a turbulence model is planned for the future. We split the electric potential ϕ, the current density $\boldsymbol{J}$ and the magnetic field $\boldsymbol{B}$ into a constant (subscript 0) and induced part (lower case) as

$$\phi = \phi_0 + \varphi \tag{2}$$

$$\boldsymbol{J} = \boldsymbol{J}_0 + \boldsymbol{j} \tag{3}$$

$$\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{b}. \tag{4}$$

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In order to determine the distribution of the constant part of the electric potential ϕ_0 we solve a Laplace equation for the electric potential

$$\nabla \cdot \sigma \nabla \phi_0 = 0 \tag{5}$$

on the global mesh. The above equation is obtained starting from the Kirchhoff law of charge conservation ($\nabla \cdot \boldsymbol{J}_0 = 0$) and $\boldsymbol{J}_0 = -\sigma \nabla \phi_0$. Note that the conductivity σ is a field and not a constant, because the equation is solved on the full geometry. During mesh generation, it is ensured that the border between two materials always coincides with a face between two neighbouring cells. The global current density is then calculated as

$$\boldsymbol{J}_0 = -\sigma \nabla \phi_0 \tag{6}$$

and mapped to the fluid region. Afterwards, the constant magnetic field is determined as described in Section 2.1.1 only in the fluid.

Often it is sufficient to calculate only the constant current and magnetic field. Nevertheless, our solver also allows to compute their induced counterparts, e.g. for simulating the Tayler instability [23–31]. The scheme is similar to that described above: in a first step, the induced electric potential φ is determined by solving a Poisson equation

$$\nabla \cdot \sigma \nabla \varphi = \nabla \cdot \sigma (\boldsymbol{u} \times \boldsymbol{B}) \tag{7}$$

after mapping the source term $\boldsymbol{u} \times \boldsymbol{B}$ to the global mesh. The induced current can be computed taking into account Ohm's law

$$\boldsymbol{j} = \sigma (-\nabla \varphi + \boldsymbol{u} \times \boldsymbol{B}). \tag{8}$$

After mapping $\boldsymbol{j}$ to the fluid mesh we determine the induced magnetic field as described in Section 2.1.1.

Our model is not capable of describing AC currents, because we use the quasi-static approximations by neglecting the temporal derivation of the vector potential ($d\boldsymbol{a}/dt = 0$) and magnetic field ($d\boldsymbol{b}/dt = 0$) [32]. For a detailed flowchart of the model, please refer to Fig. 1.

2.1.1. Computation of the magnetic field

For the computation of both, the constant part of the magnetic field $\boldsymbol{B}_0$ and its induced counterpart $\boldsymbol{b}$ we use the inversion of Ampère's law, the Biot–Savart integral

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int \frac{\boldsymbol{J}(\boldsymbol{r}') \times (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3} dV' \tag{9}$$

to determine both from the current density $\boldsymbol{J}$. This integro-differential approach was proposed by Meir and Schmidt [33–38] and later used for describing dynamos [39–41] and the Tayler instability [22].

In order to obtain the magnetic field in one single cell (at the position $\boldsymbol{r}$), the electric current densities of all other cells (at the position $\boldsymbol{r}'$) have to be integrated. The number of operations is therefore equal to the number of cells squared. This way of computation is extremely costly. We will explain here several ways for a speed up of the procedure. Solving Biot–Savart's integral on a coarser grid, recalculating it every n*th* time step, and an appropriate parallelisation [22] are the most simple ways.

The parallelisation is implemented in OpenFOAM using MPI. Basically, each processor contains only the current density of its *local* cells. With this, it computes the magnetic field for the *full* geometry (see Fig. 2a). Finally, the field $\boldsymbol{B}$ of each cell has to be summed up over all processors. This might be done using the MPI function ALLREDUCE, resulting in a correct and global $\boldsymbol{B}$ on all processors. However, this is not necessary, because a single processor needs only its *local* $\boldsymbol{B}$ for further computation. Therefore, each processor receives only its *local* magnetic field from all other processors and adds up all contributions given. The communication process is illustrated in Fig. 2b. Increasing the speed-up considerably is possible by computing Biot–Savart's integral only on the boundaries and solving the induction equations [42,43]

$$0 = \Delta \boldsymbol{B}_0 \tag{10}$$

$$0 = \frac{1}{\sigma \mu_0} \Delta \boldsymbol{b} + \nabla \times (\boldsymbol{u} \times \boldsymbol{B}_0) + \nabla \times (\boldsymbol{u} \times \boldsymbol{b}) \tag{11}$$

for the constant and induced magnetic field in the quasi-static limit [32].

An even faster alternative is shifting the problem from the magnetic field $\boldsymbol{B}$ to the vector potential $\boldsymbol{A}$ using the relation $\boldsymbol{B} = \nabla \times \boldsymbol{A}$. Similar to Biot–Savart's law for $\boldsymbol{B}$, the vector potential can be determined by Green's identity [44]:

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int \frac{\boldsymbol{J}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|} dV'. \tag{12}$$

Please note that this formula is much cheaper to compute than Biot–Savart's law (Eq. 9) [45,46].

The transport equations for the vector potential are derived from Ampère's law, $\boldsymbol{B} = \nabla \times \boldsymbol{A}$, Ohm's law [47] and using the Coulomb gauge condition $\nabla \cdot \boldsymbol{A} = 0$ as

$$0 = \frac{1}{\sigma \mu_0} \Delta \boldsymbol{A}_0 - \nabla \phi_0 \tag{13}$$

$$0 = \frac{1}{\sigma \mu_0} \Delta \boldsymbol{a} + \boldsymbol{u} \times \boldsymbol{B}_0 + \boldsymbol{u} \times (\nabla \times \boldsymbol{a}) - \nabla \varphi. \tag{14}$$

Basically all mentioned approaches of determining $\boldsymbol{B}$ based on Eq. (9) till (14) are equal from a physical point of view. But due to the way they are discretised and numerically solved, there will be differences in both accuracy and calculation time. While being the most expensive method, calculating the magnetic field by means of Biot–Savart's law also gives the most accurate result. This stems from the fact that the integral equation (9) represents an exact solution for $\boldsymbol{B}$ which is only numerically integrated for a finite number of cells. As already mentioned, a computationally less expensive evaluation can be achieved with the help of the magnetic vector potential $\boldsymbol{A}$ and Green's identity (12), where the complexity of the integrand is reduced compared to Eq. (9). Despite of Eq. (12) also being an exact solution, the subsequent calculation of $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ introduces an additional layer of discretisation errors from cell averaging and face interpolation.

As outlined above, Biot–Savart's law may be used also in combination with Eqs. (10) and (11) or Green's identity (12) combined with Eqs. (13) and (14), while only boundary values of $\boldsymbol{B}$ or $\boldsymbol{A}$ are evaluated using the exact integral equations. Internal values are then recovered from solving the related differential equations. This drastically improves computational efficiency at the cost of some accuracy. However, from Fig. 1 one can comprehend that it is sufficient to calculate $\boldsymbol{B}_0$ or $\boldsymbol{A}_0$ only once at the beginning of a simulation, whereas $\boldsymbol{b}$ or $\boldsymbol{a}$ needs to be updated recurringly while marching in time. The most promising way of determining the total magnetic field $\boldsymbol{B}$ is thus to compute its static part $\boldsymbol{B}_0$ once and solely using Biot–Savart's law with the current density $\boldsymbol{J}_0$ and the induced part $\boldsymbol{b}$ mediately from $\boldsymbol{b} = \nabla \times \boldsymbol{a}$, whereby the solution of the induced magnetic vector potential $\boldsymbol{a}$ is in turn based on the transport Eq. (14) for which Dirichlet boundary conditions are derived from evaluating Green's identity with the current density $\boldsymbol{j}$. This approach has been used for all following calculations.

In this way, $\boldsymbol{B}_0$ is most accurate and $\boldsymbol{b}$ is repeatedly calculated with minimum computational effort. Another important advantage of this realisation is that the solenoidal nature of $\boldsymbol{B}$ is implicitly satisfied, as $\boldsymbol{B}_0$ results from an exact solution in shape of Biot–Savart's law and $\boldsymbol{b}$ is calculated from the definition of the induced vector potential $\boldsymbol{a}$ with $\nabla \cdot \boldsymbol{b} = \nabla \cdot (\nabla \times \boldsymbol{a}) \equiv 0$. Numerically, Gauss's law $\nabla \cdot \boldsymbol{B} = 0$ is of course only met approximatively due