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A new interpolation technique to deal with fluid-porous media interfaces for topology optimization of heat transfer

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ABSTRACT

This paper proposes a new interpolation technique based on density approach to solve topology optimization problems for heat transfer. Natural convection forces are dominated as Richardson number is equal to 2.8. Problems are modeled under the assumptions of steady-state laminar flow using the incompressible Navier–Stokes equations coupled to the convection-diffusion equation through the Boussinesq approximation. The governing equations are discretized using finite volume elements and topology optimization is performed using adjoint sensitivity analysis. Material distribution and effective conductivity are interpolated by two sigmoid functions respectively $h_\tau(\alpha)$ and $k_\tau(\alpha)$ in order to provide a continuous transition between the solid and the fluid domains. Comparison with standard interpolation function of the literature (RAMP function) shows a smaller transition zone between the fluid and the solid thereby, avoiding some regularization techniques. In order to validate the new method, numerical applications are investigated on some geometric configurations from the literature, namely the single pipe and the bend pipe. Lastly, as two new parameters are introduced thanks to the interpolation functions, we study their impact on results of the optimization problem. The study shows that the proposed technique is a viable approach for designing geometries and fluid-porous media interfaces are well-defined.

1. Introduction

Since its introduction by Bendsoe and Sigmund [1] for solid mechanics problems, topology optimization has become a powerful and increasingly popular tool for designers and engineers for design process. Topology optimization is a material distribution method for finding the optimal structure, for a given problem subject to design constraints. Contrarily to shape optimization where the topology (i.e. the number of boundaries and connectivity) is predetermined, topology optimization allows introduction of new boundaries during the design process.

Topology optimization was pioneered for Stokes flow by Borrvall and Petersson [4]. They introduced a friction term yielding the generalized Stokes equations. Gersborg-Hansen [9] and Olesen et al. [8] extended topology optimization for fluid flow problems to the Navier–Stokes equations.

In topology optimization, the material distribution is parametrized by defining a design variable $\alpha \in \{0; 1\}$. This variable is discrete and should either represent solid material ($\alpha = 1$) or fluid ($\alpha = 0$). A common approach to solve the topology optimization problem with this discrete value as optimization parameter is to change it into a continuous one by introducing a porous media with a continuous permeability variable for each element. This method, known as the Brinkman penalization, leads to a problem where flow and (almost) non-flow regions are developed by allowing interpolation between the lower and upper value of permeability. Generally, authors used the density interpolation function proposed by Borrvall and Petersson [4] or a reformulated version of their convex and q-parametrized interpolation function. The parameter $q > 0$ is a penalty parameter that is used to control the level of 'gray' in the optimal design. However, authors had also obtained unsatisfactory optimal solutions. Therefore, they considered a two-step solution procedure where the problem was first solved with a small penalty value of $q = 0.01$ for example and then the result is used as initial case for the problem with a penalty value of $q = 0.1$ [4,8] or $q = 1$ [15]. The mathematical foundation of the interpolation of α was further investigated by Evgrafov [14] where the limiting cases of pure fluid and solid were included. Brinkman approach has since been used for several problems as transport problem [28], reactive [32] and transient flows [3,33], fluid-structure interaction [36] and also flows driven by body forces [37].

A variation of the approach is presented by Guest and Prevost [2]. They proposed to regularize the solid-fluid structure by dealing the material phase as a porous medium where fluid flow is

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Nomenclature

Physics Constants

g	Gravitational acceleration
h_τ	Ratio between a kinematic viscosity and a permeability
k_τ	Thermal conductivity, dimensionless
l	Characteristic length
p	Pressure, dimensionless
$\mathbf{u}$	Velocity, dimensionless
Gr	Grashof number
Pr	Prandtl number
Q_t	Proportion of material added in Ω at the end of optimization
Re	Reynolds number
Ri	Richardson number
T	Fluid temperature
$\mathcal{J}$	Objective function
$\mathcal{L}$	Lagrange function
$\mathcal{T}$	Size of transition zones

Greek symbols

α	Design parameter
$\alpha_{\max}$	Maximal value that h_τ can reach
α_0	Parameter of sigmoid function
β	Thermal expansion coefficient of fluid
Γ	Boundary of domain Ω
ϵ	Stopping criterion in optimization algorithm
θ	Temperature, dimensionless
λ_f	Thermal diffusivity of the fluid
λ_s	Thermal conductivity of the solid
τ	Parameter of sigmoid function
Φ	Constant heat flux
Ω	Spatial domain

Subscripts and other symbols

0	Initial condition
n	Normal component of vector
t	Tangential component of vector
in	Relative to the inlet
out	Relative to the outlet
R	Relative to RAMP function
S	Relative to Sigmoid function
$S, 0$	Relative to Sigmoid function with $\alpha_0 = 0$
$*$	Relative to Lagrange multipliers or adjoint variables

governed by Darcy's law. In their approach, flows through voids are governed by Stokes flow and, when the solid phase is impermeable, discrete no-slip condition is simulated by assigning a low permeability to the solid phase. There exists other alternatives to Brinkman penalization in the literature. The level set approach to topology optimization has been applied to fluid flows problems [13,31,34,40,44], and recently the level set approach was combined with the extended finite element method (XFEM) by Kreissl and Maute [5] and by Jenkins et al. [39]. The main drawback of the level set approach is the constraint of remeshing throughout the optimization process.

The second difficulty in topology optimization is to deal with the difference in thermal conductivity in the solid and fluid domains. Most publications interpolate the conductivity using the SIMP method (Solid Isotropic Material with penalization). This method allows to deal with the discrete nature of conductivity material distribution. So, authors [6,15,30,36] considered a continuous local thermal conductivity controlled by the design parameter α ranging from 0 to 1. Thanks to this function, the optimization algorithm is able to reallocate thermal conductivity material, or creating 'holes' in its structure to reach the objective function. Moreover, the convex and q-parametrized function interpolation is similar to the density interpolation function of Borrvall and Petersson [4] or the RAMP (Rational Approximation of Material Properties)-style function as introduced by Stolpe [7,16,30]. Other methods have also been investigated. Matsumori et al. [19] presented results with a linear-interpolated design-dependant volumetric heat generation. Dede [20] used a linear interpolation for thermal conductivity. Thus, the main issue is to deal with intermediate design variables and non-physical flow solutions. That corresponds to numerical solutions where the fluid has a non-null velocity in a region assigned to solid regions. We can also obtain optimized designs with zones containing a fluid flow although there is no path from the inlet for the flow.

There are three main categories of algorithms to solve topology optimization problems : gradient-free, gradient-based and hessian-based algorithms. In topology optimization problems with large number of design variables, gradient-based algorithms are used to find accurate solutions efficiently. One of the advantages of the interpolation functions described above is the possibility of using gradient-based continuous optimization methods. These methods are based on derivatives in order to find extrema, and are the so-called sensitivity analysis. It aims at evaluating the derivative of objective function with respect to α. Gradient-based algorithm is widely used by several authors [21,27,30,35,45]. Moreover, since most of the topology optimization problems involve a huge amount of design variables, specific gradient-based optimization algorithms must be chosen to handle this difficulty. A famous algorithm from the literature is the MMA (Method of Moving Asymptotes) developed initially by Svanberg[41]. In order to reduce the computational costs, adjoint approach consisting to calculate the sensitivities of the objective function by an adjoint state has been adopted. Other methods have been explored to reduce the computation cost: the multigrid preconditioned conjugate gradients (MGCC) by Amir et al. [22], multi-resolution multi-scale topology optimization technique by Kim et al. [23], the technique of using adaptive design variable fields by Guest et al. [26].

Moreover, various regularization techniques based on filtering of either the design variable α or the sensitivity $\frac{\partial f}{\partial \alpha}$ [1,10,15,30,46] exist to ensure well-posed topology optimization problems. The regularization works by defining a certain length scale r_0 below which any features in α or $\frac{\partial f}{\partial \alpha}$ are smeared out by the filter; that results in optimized structures with a minimal feature size r_0 independent of the mesh refinement. As mentioned by some authors [15,29], these regularization techniques allow to avoid checkerboard problems.

This paper proposes a new interpolation technique in order to solve a heat transfer topology optimization problem. Design material and effective conductivity are interpolated respectively by a function $h_\tau(\alpha)$ and another function $k_\tau(\alpha)$ in order to provide a continuous transition between the solid and the fluid domains. These interpolation functions avoid the use of some regularization techniques because the problem can be solved in one-step without a new value of the convexity parameter. Moreover, these interpolation functions allow a smaller transition zone between the fluid and solid regions. To prove this claim and get more qualitative results, the size of these transition zones is explicitly computed and comparison with standard RAMP interpolation is led. In order to validate the new method, some numerical applications are investigated on the single pipe and the bend pipe cases. Lastly, as two new parameters are introduced thanks to the interpolation functions, we study their impact on the results of an optimization problem.

The main novelty of this paper is first the definition of a new interpolation function that avoids the use of some regularization