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A multiphase level-set approach for all-Mach numbers

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ABSTRACT

In this work, an alternate level-set-based approach is presented that applies uniformly to compressible and incompressible multiphase flows. Fundamental to this work, is the development of analytic transformations from a signed-distance function to species-mass conservation variables. Such transformations can be used to highlight compressible flow difficulties for level set methods, and develop interfacial reinitialization procedures based on different primitive variables. The proposed all-Mach method is based on preserving signed-distance functions within the context of a species-mass conservation equation to evolve the interface, and includes several reinitialization procedures that maintain the spirit of the signed distance function. In addition, we explore hybrid level-set reinitialization procedures that handle sub-grid-scale interfacial breakup. The model is demonstrated on concepts relevant to high-speed marine vehicles based on supercavitation, where a gaseous cavity surrounds a moving vehicle. Results indicate that the present algorithm preserves higher-order numerics, performs well on several incompressible and compressible validation cases, and extends to unsteady, three-dimensional flow.

1. Introduction

The level-set method [1] is one technique for compressible and incompressible multiphase fluid flow modeling. Its usage is attractive as it handles complex material fronts in a natural and general way with the use of a scalar function that evolves with the local flow. Using this scalar function, the method implicitly defines an evolving interface within the numerical model. Perhaps the principal benefit of level set methods is their ability to model nondiffusing interfaces. The methods have been applied to a broad number of applications [2,3] including bubble dynamics [[4–8], etc.], multiphase flow instabilities [9], shocks [2,9–12], free-surface flows [13–16], and others [2,3]. These are only a small number, of the many multiphase flow problems that require non-diffusing interfaces.

The goal of this work is to extend the level-set method to a high-speed, underwater vehicle concept referred to as supercavitation. In supercavitation, gaseous cavities are formed by ventilating gas into a liquid flow such that the vehicle is fully enclosed by gas. Such multiphase flow problems are characterized with regions of both compressible and incompressible flow, and contains regions with a well-defined interface (suitable for level-set methods) combined with frothy regions (not suited for level-set methods with a coarse mesh). These aspects are encountered and treated with a new level-set methodology.

In this paper, a level-set method that addresses the issues described above is developed and tested against a range of relevant test problems. The present method is based on evolving a species-mass conservation equation, rather than the more traditional signed-distance function, which we show are analytically equivalent in the incompressible limit. In aligning with a species-mass-conserving (SMC) multiphase-fluid-flow formulation [17–26] our method displays consistency with the governing equations in all flow regimes. The approach varies from other SMC formulations in that the non-diffusing interface, characteristic to level-set methods, is recovered. Results from the approach indicate that the method alleviates oscillations in compressible flow, enables a level-set method that can adapt to sub-grid scale mixtures (relevant when the computational mesh is to coarse to resolve the interface), and provides a simple method to implement a level-set methodology in a SMC formulation.

The present work is an evolution and refinement of previous work presented in Kinzel [27] and Kinzel et al. [28] and is organized as follows. In Section 2, a brief review of the governing equations for homogenous multiphase flows and level-set methods. Section 3 introduces a novel transformation between a signed-distance function to a volume fraction that has an analytic inverse. The transformation is used to evaluate traditional level-set methods with respect to the governing equations, and we highlight various inconsistencies. The transformations also display an approach

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Abbreviations

Symbols

a	speed of sound
C_p	specific heat at constant pressure
C_v	specific heat at constant volume
D_N	cavitator diameter
e	internal energy
f_{ls}	sharpening frequency
f_d	relaxation of reinitialization
F	flux vector
F_{DES}	detached eddy simulation parameter
Fr_N	Froude number based on cavitator diameter, $V_\infty(gD_N)^{-0.5}$
g	gravitational acceleration constant
h_{lg}	latent heat of vaporization
H	Heaviside function
k_ϕ	signed-distance function approximation parameter
M	Mach number, V/a
n	number of species
p	pressure
Pr	Prantl number
q	dynamic pressure, $1/2\rho V^2$
Q	ventilation rate
Re	Reynolds number, $\rho VL/\mu$
t	time
t_{ij}	viscous stress tensor
T	temperature
u	velocity in Cartesian direction
V	velocity
Y	mass fraction

Greek symbols

δ	dimensional cell length
ε	interface thickness in level-set method
ε_2	sharpened volume fraction in level-set method
χ	regularized characteristic function
γ	ratio of specific heats, c_p/c_v
κ	thermal conductivity
ϕ	signed-distance function
μ	molecular viscosity
ρ	density
σ_v	cavitation number based on vapor pressure, $(p_\infty - p_v)/q_\infty$
ω	mass transfer rate

Superscripts

g	references gaseous species
L	references liquid species
m	iteration index
s	phase index

Subscripts

c	reference to cavity properties
D	reference to diameter
m	reference to mixture quantity
max	reference to a maximum
min	reference to a minimum
N	reference to cavitator
t	reference to turbulent quantity
v	references vapor
∞	reference to free stream

GFPM	Ghost Fluids for the Poor Method
SMC	Species-Mass Conservation
SDF	Signed-Distance function

to reinitialize a volume-fraction field based on the signed distance function. In Section 4, several new reinitialization strategies for a volume-fraction field are presented, two of them based on replicating a signed distance function, and the third is a hybrid level-set method that is applicable to conditions where interface break up occurs. In Section 5, the level set model is presented within the all-Mach number formulation developed by Lindau et al. [20,21]. Finally, in Section 6, the methods are tested on simple and complex, compressible and incompressible, multiphase-flow problems.

2. Multiphase flows and level-set methods

A brief introduction to the governing equations for a homogeneous multiphase fluid flow is given and evaluated with respect to various level set approaches.

2.1. Governing equations for homogeneous-multiphase flow

The governing equations for a compressible, viscous, homogenous-multiphase flow, are presented in Cartesian-tensor notation as:

$$\frac{\partial Q_c}{\partial t} + F_{j,j} - F^{v}_{j,j} = H. \tag{1}$$

The conservative variables, Q_c, are defined as

$$(Q_c)^T = \left[\rho \ \ \rho u_i \ \ e \ \ (\tilde{\rho}\alpha)^s\right]. \tag{2}$$

Here ρ and e are local mixture quantities of density and internal energy, respectively, which are computed using the mixing rules defined by Amagat's Law. For example, using the species volume fraction, α^s, and the species-specific density, $\tilde{\rho}^s$, the mixture density is computed as

$$\rho = \sum_{s=1}^{n} \alpha^s \tilde{\rho}^s, \tag{3}$$

and the mixture internal energy by

$$e = \sum_{s=1}^{n} \alpha^s \tilde{\rho}^s \tilde{e}^s. \tag{4}$$

The inviscid-flux vector is defined as

$$\left(F_j\right)^T = \left[\rho u_j \ \ \rho u_i u_j + p\delta_{ij} \ \ u_j(e+p) \ \ (\tilde{\rho}\alpha)^s u_j\right] \tag{5}$$

and the viscous-flux vector is defined as

$$\left(F^v_j\right)^T = \left[0 \ \ t_{ij} \ \ \left(u_k t_{kj} + \frac{\kappa_{m,t}\partial T}{\partial x_j}\right) \ \ 0\right]. \tag{6}$$

where t is the viscous stress tensor, $\kappa_{m,t}$ is the mixture (and turbulent) conductivity, and T is the temperature.

$$(H)^T = [0 \ \ \rho g_i \ \ 0 \ \ 0]. \tag{7}$$

The first elemental equation (row), in Eq. (1), represents mass conservation of the mixture. The next two rows are momentum and energy equations for the mixture. The last elemental row represents the individual SMC equations, where each superscript denotes a species number and n represents the number of species present (in this work $n = 2$).

The SMC equation may be formulated using several different variables, such as concentration, mass fraction, or volume fraction. The SMC equation is isolated below and written in terms of a liquid volume fraction, giving

$$\frac{\partial \rho^L \alpha^L}{\partial t} + \nabla \cdot \left(\vec{V}\rho^L\alpha^L\right) = 0. \tag{8}$$