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Unsteady three-dimensional boundary element method for self-propelled bio-inspired locomotion

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ABSTRACT

An unsteady three-dimensional boundary element method is developed to provide fast calculations of biological and bio-inspired self-propelled locomotion. The approach uniquely combines an unsteady three-dimensional boundary element method, a boundary layer solver and self-propelled equations of motion. This novel implementation allows for the self-propelled speed, power, efficiency and economy to be accurately calculated. A Dirichlet formulation is used with a combination of constant strength source and doublet elements to represent a deforming body with a nonlinearly deforming wake. The wake elements are desingularized to numerically stabilize the evolution of the wake vorticity. Weak coupling is used in solving the equations of motion and in the boundary layer solution. The boundary layer solver models both laminar and turbulent behavior along the deforming body to estimate the total skin friction drag acting on the body. The results from the method are validated with analytical solutions, computations and experiments. Finally, a bio-inspired self-propelled undulatory fin is modeled. The computed self-propelled speeds and wake structures agree well with previous experiments. The computations go beyond the experiments to gain further insight into the propulsive efficiency for self-propelled undulating fins. It is found that the undulating fin produces a time-averaged momentum jet at 76% of the span that accelerates fluid in the streamwise direction and in turn generates thrust. Additionally, it is discovered that high amplitude motions suppress the formation of a bifurcating momentum jet and instead form a single core jet. Consequently, this maximizes the amount of streamwise momentum compared to the amount of wasted lateral momentum and leads to a propulsive efficiency of 78% during self-propelled locomotion.

1. Introduction

Boundary element methods (BEMs) are a class of numerical methods used to solve boundary value problems throughout physics from electromagnetics [22] and fracture mechanics [36] to fluid flows at both low [37] and high Reynolds numbers [3]. In high Reynolds number flows they are classically described as panel methods and have been well established in the study of aerodynamics over several decades [20,24,29]. High Reynolds number BEMs assume that a fluid flow is incompressible, irrotational (except at singular elements) and inviscid, that is, a potential flow. This leads to simplified forms of the continuity and momentum equations that govern the fluid flow. Yet, unsteady BEM solutions are still rich with flow physics [39] and give accurate solutions at computational times that are several orders of magnitude faster than Navier–Stokes solvers [35,53].

Unsteady three-dimensional BEM computations have been used by many researchers to explore both biological and bio-inspired propulsion. The flight performance of birds [47] and the swimming performance of fin whales [27] and fish [9] have been examined to reveal features of high efficiency locomotion. For example, Zhu et al. [58] found that constructive or destructive interactions can occur between the shed vorticity from finlet structures and the caudal fin of tuna and giant danio. This can lead to enhanced thrust production or efficiency, respectively, with maximum efficiencies of 75% being calculated. More recently, Zhu [56] showed that spanwise and chordwise flexibility can enhance both thrust production and efficiency of a flapping wing. The benefit of flexibility was also found to be highly dependent upon the mass ratio between the wing and the surrounding fluid environment. Additionally, Zhu and Shoele [57], Shoele and Zhu [45,46] determined that the flexibility of ray-finned fish caudal and pectoral fins also improved their efficiency performance and reduced the time-varying lateral forces acting on the fish. Importantly, none of these previous studies have examined the locomotion of *self-propelled* swim-

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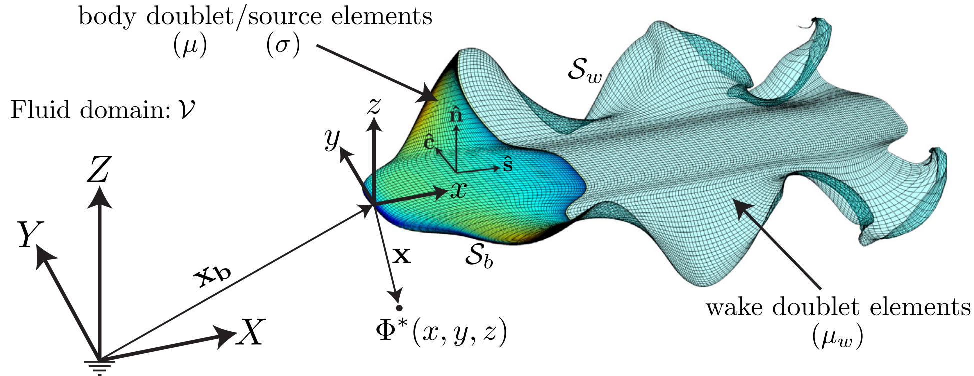

Fig. 1. The inertial reference frame fixed to the undisturbed fluid is denoted by (X, Y, Z) while the body-fixed reference frame is denoted by (x, y, z). The local normal, streamwise and cross-stream unit vectors are denoted by $\hat{\mathbf{n}}$, $\hat{\mathbf{s}}$, and $\hat{\mathbf{c}}$, respectively. The body surface, $\mathcal{S}_b$, is layered with distributions of doublet elements of strength μ and source elements of strength σ. The wake surface, $\mathcal{S}_w$, is layered with distributions of doublet elements of strength μ_w.

mers nor the *free-flight* of flyers, yet these conditions are a critical feature of bio-inspired locomotion.

One complicating factor is that an inviscid BEM does not inherently calculate viscous drag. This gives no opposing force to balance the thrust production, which leaves out a necessary ingredient for calculating a steady-state self-propelled speed. However, viscous drag has been estimated in several other BEM studies by using a boundary layer momentum-integral approach on streamwise strips [28,41,50]. Even with a viscous drag estimate included these studies focused on fixed freestream velocity conditions.

This work describes a novel implementation for computing the self-propelled performance of biological and bio-inspired propulsors within a BEM framework. There are three main components that must be combined to model self-propelled swimming: (1) a three-dimensional BEM fluid solver, (2) a boundary layer solver, and (3) an equations of motion solver. These components to the method are described in Section 2. Validation with several analytical, numerical and experimental solutions are presented in Section 3. Finally, comparison of the BEM solution with a three-dimensional self-propelled undulating fin experiment is presented in Section 4. The free-swimming performance and wake structures are shown to agree well with the experiments. Additionally, the self-propelled performance of cases that extend beyond the previous experiments are examined to provide novel physical insight into the self-propulsion of three-dimensional ray-inspired fins.

2. Computational methods

2.1. Governing equations and boundary conditions

To model a high Reynolds number fluid flow around a self-propelled bio-inspired device or animal an unsteady three-dimensional boundary element method is employed. The flow field is modeled as an incompressible, irrotational and inviscid flow, that is, a potential flow. For the self-propelled problem we define the problem in an inertial frame of reference that is attached to the undisturbed fluid (denoted by (X, Y, Z) in Fig. 1). As such the velocity field, $\mathbf{u}$, may be defined everywhere as the gradient of a scalar velocity potential,

$$\mathbf{u} = \nabla \Phi^*, \tag{1}$$

where Φ^* is defined in the inertial frame of reference and it is known as the perturbation potential. The pressure field, P, within this fluid can be calculated from the unsteady Bernoulli equation,

$$P(X, Y, Z, t) = -\rho \frac{\partial \Phi^*}{\partial t}\bigg|_{inertial} - \rho \frac{\left(\nabla \Phi^*\right)^2}{2}, \tag{2}$$

which is formulated in the inertial frame where the reference pressure $P_\infty = 0$ and the perturbation potential at infinity is zero. Also, ρ is the fluid density. The time derivative of the perturbation potential for a point on the surface of the body is then calculated by using a body-fixed Lagrangian frame (denoted by (x,y,z) in Fig. 1) [9,35,54], that is,

$$P(x, y, z, t) = -\rho \frac{\partial \Phi^*}{\partial t}\bigg|_{body} + \rho(\mathbf{u_{rel}} + \mathbf{U_0}) \cdot \nabla \Phi^* - \rho \frac{\left(\nabla \Phi^*\right)^2}{2}. \tag{3}$$

The translational velocity of a body-fixed frame of reference is $\mathbf{U_0}$ while the relative velocity of a point on the surface of the body to the body-fixed reference frame is $\mathbf{u_{rel}}$. Once the perturbation potential is known, then the pressure on the body surface may be found and the forces can be calculated by integrating the pressure and shear stress, τ, acting on the body.

$$\mathbf{F}(x, y, z, t) = \int_{\mathcal{S}_b} \left(-P\,\hat{\mathbf{n}} + \tau\,\hat{\mathbf{s}}\right) d\mathcal{S} \tag{4}$$

The body surface is denoted as $\mathcal{S}_b$, the outward normal vector from the body surface is $\hat{\mathbf{n}}$ and the tangential vector along the body surface in the streamwise direction is $\hat{\mathbf{s}}$. This inviscid formulation is coupled to a viscous boundary layer solver described in Section 2.9, which estimates the shear stress acting on the body in the streamwise direction produced by the outer potential flow. Note that the shear stress acting in the cross-stream direction is not accounted for in the viscous boundary layer solver and is therefore not present in Eq. (4). The problem is then reduced to solving for the perturbation potential throughout the fluid, which is governed by Laplace's equation,

$$\nabla^2 \Phi^* = 0. \tag{5}$$

The boundary conditions that must be satisfied for an inviscid fluid are that there is no fluid flux through the body surface and that the flow disturbances caused by the body must decay far away,

$$\mathbf{n} \cdot \nabla \Phi^* = \mathbf{n} \cdot (\mathbf{u_{rel}} + \mathbf{U_0}) \qquad \text{on } \mathcal{S}_b \tag{6}$$

$$\nabla \Phi^*\Big|_{|\mathbf{x}| \to \infty} = 0 \qquad \text{on } \mathcal{S}_\infty \tag{7}$$

where $\mathcal{S}_\infty$ is the surface at infinity bounding the fluid and $\mathbf{x} = [x, y, z]^T$ is measured from the body-fixed frame of reference.

2.2. Boundary integral equation

A general solution to Laplace's equation for the potential anywhere within the fluid domain, $\mathcal{V}$, can be determined. This is done