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Benchmark solutions

Simulating flow over and through porous media with application to erosion of particulate deposits

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ABSTRACT

We simulate flows involving porous media and homogenous fluid using a single-domain finite-difference numerical method. The porous medium and unimpeded fluid are separated by a sharp interface where a stress jump boundary condition is implemented using a forcing term. The interface is constructed by connecting Lagrangian markers with cubic splines, allowing for any possible porous media geometry. This model is particularly flexible as it can easily account for a mobile interface. We apply our method to simulate erosion and suspension of particles from a fixed or erodible particulate deposit. The flux of particles entrained from the porous media is obtained from the computed velocity at the interface, in contrast to more common approaches that assume a flux proportional to the viscous stress at the interface.

1. Introduction

In many natural settings and industrial applications, fluid flows in an environment partially consisting of porous medium. Examples where this setup occurs include rivers and oceans flowing over sandy deposits [1–4], oil reservoirs and groundwater flow [5], flow past porous scaffolds in bioreactors [6] and blood clot formation [7–9]. In particular, the chemical pollution of our water resources through contaminated deposits is a significant issue [10], which requires models that can predict and quantify the spread of chemicals removed from contaminated soil by rainfall and flooding [11]. For most geometries, only numerical solutions are available. We present here a numerical method to simulate a system consisting of porous media and homogenous fluid separated by a sharp interface. This method is simple to implement and allows for a mobile interface between the porous and fluid regions.

The interaction of a freely flowing fluid with a porous matrix is complex and considerable effort has been dedicated to determine proper treatment of the interface between the two media. One of the early studies of the fluid–porous boundary condition was done by Beavers and Joseph in 1967 [12], where a semi-empirical slip velocity corresponding to a velocity jump was introduced to match the Navier–Stokes equations with a porous flow described by Darcy's law. Neale and Nader [13] used the Brinkman equations to describe the porous flow, and assumed continuous stress and velocity across the interface. The use of the Brinkman's equations and its associated effective viscosity, has been used by numerous authors [14–16] since it allows for more accurate matching at the interface. Vafai and Kim presented an exact analytical solution for fluid flow at the interface by matching both velocity and stress [17]. A more detailed volume-averaging study led by Ochoa-Tapia and Whitaker [18] deduced a condition of tangential stress jump at the interface which has since been widely accepted and used in many applications [19–21]. We will therefore make use of the Brinkman equations to describe the flow in the porous medium, and of a tangential stress jump boundary condition at the interface, for more accurate results.

Fluid–porous problems are typically solved either using a two-domain approach [16,22], or a single-domain approach [23–25]. In a two-domain approach, two sets of coupled governing equations are applied to the fluid and porous regions of the domain and matching boundary conditions are enforced at the interface. This approach is more complicated to implement but provides means to apply a broad range of boundary conditions at the fluid–porous interface [26]. In a single-domain approach the entire domain is represented by one governing equation with variables undergoing a spatial variation across the interface [27]. This formulation avoids the explicit matching of boundary conditions at the fluid–porous interface and is widely used in numerical simulations of thermal natural convection [28]. More recently, it was also applied to describe the saturation of porous media by a liquid [25]. We

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will make use of this approach to apply a tangential stress-jump boundary condition at the fluid–porous interface.

Various numerical methods have been used to solve the equations governing flow in fluid–porous domains. Because of the complex geometries associated with porous media, finite elements have been widely used [19–21,29] as have finite volumes [26]. However many of these approaches rely on a fixed boundary between the two media. Our proposed method is based on finite-difference approximations of derivatives and accurately describes the flow in both media while allowing for a moving interface between the two sub-domains. The potentially mobile interface is defined by Lagrangian markers, whose position is governed by a simple differential equation. The stress-jump boundary condition is applied through a forcing term along the interface without requiring any matching. This approach is analogous to the immersed boundary method [30] and related methods used in multiphase flow [31–33]. There, a stress jump is imposed across a fluid–fluid interface, and the Navier–Stokes equations are solved on both sides of the interface with different parameters. Here, we employ a similar approach, but solve the Brinkman equations on one side of the interface, and the Navier–Stokes equations on the other. The immersed boundary method is often implemented using numerical delta functions to capture the interfacial forcing while in multiphase fluid flow a Volume-Of-Fluid method is often used, which corresponds to an interface that is one grid-cell thick. Here, we will compare both implementations.

After validating our method, we proceed to use it to describe the motion of fluid and particles in erosive systems where the geometry of the surface over which currents propagate may undergo a continuous change. At present, a complete understanding of resuspension of particles from an irregular bed of particles remains elusive. Direct numerical simulations have been employed to study the lift-off of particles in plane Poiseuille flow [4], but such simulations are constrained to a limited number of circular particles [34]. In contrast, the continuum approach used here may describe much larger systems. Our fluid–porous solver can also be used as part of a larger fluid solver to locally quantify suspension of particles and predicting changes in surface geometry. Suspended particles representing different type of contaminants, viruses or bacteria, could also easily be traced by adding an advection–diffusion equation and tracking concentration fields [35].

The remainder of this paper is organized as follows. We first present the governing equations in Section 2. Our model and numerical approach are described in Section 3 and validated in Section 4. We discuss our results and consider an application to erosion with a fixed interface in Section 5 and with a moving interface in Section 6. Finally, we present our conclusions in Section 7.

2. Governing equations

We consider an incompressible Newtonian fluid flowing over a fixed porous medium with potentially variable permeability, see Fig. 1. We make use of the continuum approach, as opposed to a model tracking individual particles, to allow for simulations over domains encompassing more particles. The homogenous fluid portion of the system is governed by the incompressible Navier–Stokes equations

$$\nabla \cdot \vec{u} = 0 \tag{1}$$

$$\rho\left(\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\nabla\vec{u}\right) = \nabla\cdot\bar{\bar{T}}_f + \vec{f} \tag{2}$$

where $\bar{\bar{T}}_f = -p\bar{\bar{I}} + \mu(\nabla\vec{u} + (\nabla\vec{u})^T)$ is the fluid stress tensor, $\vec{u}$ the fluid velocity vector, ρ the fluid density, p the pressure field, μ the fluid viscosity, and $\vec{f}$ a generic body force.

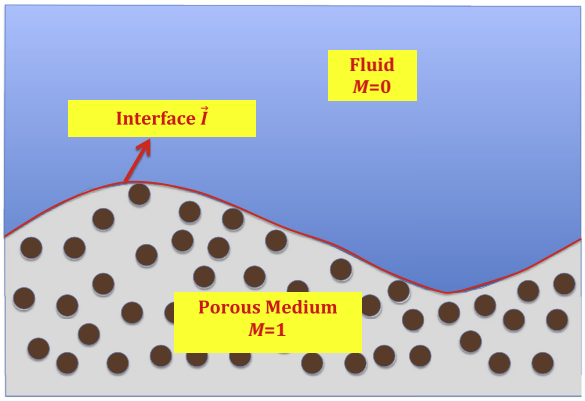

Fig. 1. Schematics of the domain under consideration. The porous medium (below) and homogenous fluid (above) are separated by a sharp interface $\vec{I}$. Flow in the fluid portion is described by the Navier–Stokes equations, and flow within the porous medium is described by the Brinkman equations.

The equations used to describe the flow within porous media typically depends on the specific flow characteristics such as the local Reynolds number, porosity, and accuracy level desired in the vicinity of the boundaries. We assume here a stationary porous matrix, saturated by a fluid flowing with a locally low Reynolds number. Under these conditions, the flow adjusts instantaneously to changes in the boundary conditions and is well described by the Brinkman equations [20]

$$\nabla \cdot \vec{u} = 0 \tag{3}$$

$$\mu\vec{u} = \bar{\bar{\kappa}}(\nabla\cdot\bar{\bar{T}}_p + \vec{f}) \tag{4}$$

where $\bar{\bar{T}}_p = -p\bar{\bar{I}} + \mu_e(\nabla\vec{u} + (\nabla\vec{u})^T)$ is the stress tensor in the porous domain, μ_e is the effective viscosity inside the porous medium, $\bar{\bar{\kappa}}$ is the permeability tensor, and $\vec{f}$ is again a generic body force. We note that the Brinkman equations are of second order in space, like the Navier–Stokes equations, which will facilitate matching at the interface, $\vec{I}$, separating the two sub-domains.

For configurations containing both fluid and porous medium, boundary conditions have been derived to match flow quantities at the interface between the two sub-domains. We enforce the commonly used boundary condition of continuous velocities and a tangential stress jump [18] across the interface $\vec{I}$, which result from balancing mass, momentum and energy [16]. So at the interface we have:

$$\vec{u}_p = \vec{u}_f = \vec{u}_{\vec{I}} \tag{5}$$

$$\vec{n}\cdot(\bar{\bar{T}}_f - \bar{\bar{T}}_p)\cdot\vec{t} = \frac{\zeta\mu}{\sqrt{K}}\vec{u}_{\vec{I}}\cdot\vec{t}, \tag{6}$$

where $\vec{t}$ and $\vec{n}$ are, respectively, unit tangential and normal vectors to the interface, ζ is the stress jump coefficient, an empirical constant of order 1 and $K = ||\bar{\bar{\kappa}}||$ is the magnitude of the permeability tensor, taken for example as the square root of the sum of the squares of components of $\bar{\bar{\kappa}}$ (Frobenius norm). We note that the tangential velocity is only differentiable at the interface when there is no stress jump, i.e. when $\zeta = 0$.

To model both the fluid and porous regions, we adopt here a single domain approach using a convex combination of the Brinkman and Navier–Stokes equations. To distinguish between the two domains, we use an indicating function M, which is set to 0 in the homogenous fluid and to 1 in the porous medium, as shown in Fig. 1. Rather than solving separately the governing equations in both domains and matching them using (6), we impose the stress jump boundary condition by the addition of an interfacial force, $\vec{f}_{\vec{I}}$,