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# A double-distribution-function lattice Boltzmann model for high-speed compressible viscous flows

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ABSTRACT

A lattice Boltzmann model for high-speed compressible viscous flows is presented based on the double-distribution-function lattice Boltzmann method proposed by Li et al. (2007). The D2Q17 circle function is introduced to take into account first to fourth order constraints of density equilibrium distribution function, in order for better consistency in the heat flux and the energy dynamics. The corresponding total energy equilibrium distribution function is formed. The present model is tested through three problems, i.e., the Riemann problem, regular shock reflection problem and supersonic boundary layer problem. We also observe improved performance of the new model for a supersonic boundary layer problem in comparison to the original coupled double-distribution-function lattice Boltzmann method.



## 1. Introduction

Recently, the lattice Boltzmann method (LBM) [1,2], a mesoscopic computational fluid dynamic (CFD) method, has become an outstanding numerical method for fluid flows. Different from traditional CFD methods which are addressing the direct discretization of the Navier–Stokes equation, the LBM focuses on the evolution of particle clusters. In fact, all macroscopic fluid flows emerge from the collective dynamics of large particle ensembles. The LBM shows great potential for complex phenomena due to its mesoscopic features and highly efficient parallel computing [3–5]. Due to these special features, the LBM has been employed to simulate various complex flows successfully, such as flows in porous media [6,7], electro-osmotic flow [8], multiphase flows [9–12], multicomponent flows [13,14], etc. In recent years, LBM is also applied to many kinds of compressible flows, such as shock waves [15,16], Richtmyer–Meshkov Instability [17], Rayleigh–Taylor instability [18], combustion and detonation [5], etc. However, most applications of compressible LBM focus on inviscid cases. It is hard to see LBM's application in compressible viscous flows. In order to advance the development of LBM for engineering applications, LBM should also be extended to reliably model compressible viscous flows.

The compressible flows are ubiquitous in aerophysics, astrophysics, explosion physics, and other areas. It is also a kind of basic flow in aerospace engineering. In fact, the theory of compressible LBM is still under development, which limits its application. Alexander et al. [19] presented a muti-speed LB model only for nearly isothermal compressible flows. Yan et al. proposed a model to recover the Euler equation by using more than one rest energy levels [20]. Kataoka and Tsutahara proposed a compressible model for Navier–Stokes equations, which has an adjustable specific-heat ratio [21]. Watari also proposed a model for Navier–Stokes with flexible specific heat ratio [22]. Nevertheless, most LB models above are within the low Mach number limit.

To address this limitation, several studies have attempted to develop high-speed LB models. Sun proposed an adaptive LB model where the particle velocities vary with the Mach number and internal energy [23]. The model partly frees the particle velocity from fixed values. Yu et al. presented a LB model that has the capability of simulating compressible flows with high Mach numbers up to 5.0 [24]. The basic idea of this model is to introduce an attractive force, which effectively softens the sound speed and the Mach number is raised remarkably. Nonetheless, this is an isothermal model, its application is limited. Qu et al. proposed a LB model with a circular function in the phase field to replace the conventional Maxwellian function as the equilibrium density distribution function [25]. They formulated a D2Q13 circle function, which can satisfy all statistical relations needed to recover the Euler equations. The simple circular function is effective for LBM for high Mach number. In fact, the D2Q13 circle function also can satisfy statistical relation needed to recover compressible Navier–Stokes equations including the pressure tensor and the momentum dynamics. Li et al. developed a coupled double-distribution-function

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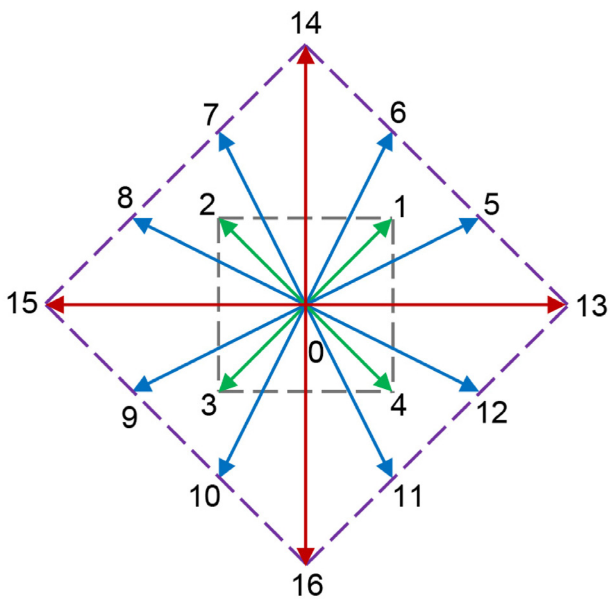


**Fig. 1.** Discrete velocities of the D2Q17 model.

(DDF) LB model for compressible flows with adjustable specific-heat ratio and Prandtl(Pr) number, and introduced the D2Q13 circle function into the density equilibrium distribution function for high-speed need [26]. Wang et al. also developed a DDF-LB model for compressible viscous flows with flexible specific-heat ratio and Pr number with a different total energy distribution function [27]. Li and Zhong presented another kind of DDF-LB model with an extra potential energy distribution function [28]. Gan et al. presented a high-speed compressible LB model by introducing an additional artificial viscosity [16]. Chen et al. proposed a multiple-relaxation-time LB model for compressible flows [29,30], which contains more physical information and has better numerical stability and accuracy than its single-relaxation-time version. Gan et al. presented a lattice Bhatnagar–Gross–Krook (BGK) kinetic model, which is a simple and general approach to formulate the lattice BGK model for high-speed compressible flows [31]. The approach is suitable for constructing LB models in any dimensions and the choice of discrete velocity model has a high degree of flexibility.

Among the high-speed LB models mentioned above, the DDF-LB model proposed by Li et al. [26] is an attractive approach, since it has only one free parameter, which is easy to implement. Besides, this model can simulate fluid flows with flexible specific-heat ratios and Pr numbers. However, according to the non-equilibrium statistical physics, the local flow density, momentum and energy should be described by the same distribution function. To reduce this potential inconsistence of the DDF model, the D2Q17 circle function is introduced to replace D2Q13 circle function to take into account first to fourth order constraints of density equilibrium distribution function. The corresponding total energy equilibrium distribution function is formed. The implicit–explicit(IMEX) finite-difference scheme [32–34] is adopted to solve the discrete Boltzmann BGK equation. The present LB model is tested through the Riemann problem, regular shock reflection problem and supersonic boundary layer problem.

## 2. Numerical method

### 2.1. Coupled DDF-LB approach

The coupled DDF-LB approach proposed by Li et al. [26] has two discrete Boltzmann BGK equations for density and total energy, respectively:

$$\frac{\partial f_\alpha}{\partial t} + (\mathbf{e}_\alpha \cdot \nabla) f_\alpha = -\frac{1}{\tau_f}\left(f_\alpha - f_\alpha^{eq}\right), \tag{1}$$

$$\frac{\partial h_\alpha}{\partial t} + (\mathbf{e}_\alpha \cdot \nabla) h_\alpha = -\frac{1}{\tau_h}\left(h_\alpha - h_\alpha^{eq}\right) + \frac{1}{\tau_{hf}}(\mathbf{e}_\alpha \cdot \mathbf{u})\left(f_\alpha - f_\alpha^{eq}\right), \tag{2}$$

where the subscript $\alpha$ represents the lattice velocity direction, $f_\alpha$ and $h_\alpha$ are the density and total energy distribution functions in the direction $\alpha$, respectively, $f_\alpha^{eq}$ and $h_\alpha^{eq}$ are their equilibrium distribution functions, $\mathbf{e}_\alpha$ is the discrete particle velocity, $\tau_f$ and $\tau_h$ are the density and energy relaxation times, and $\tau_{hf}$ is defined as $\tau_{hf} = \tau_h \tau_f / \left(\tau_f - \tau_h\right)$, $\mathbf{u}$ is the macroscopic velocity. The Pr number of the system is defined by the two relaxation times as $\mathrm{Pr} = \tau_f / \tau_h$.

To recover the compressible Navier–Stokes equations including the diffusion term of the momentum and energy equations, $f_\alpha^{eq}$ should satisfy the following velocity moment conditions:

$$\sum_\alpha f_\alpha^{eq} = \rho, \tag{3a}$$

$$\sum_\alpha f_\alpha^{eq} e_{\alpha i} = \rho u_i, \tag{3b}$$

$$\sum_\alpha f_\alpha^{eq} e_{\alpha i} e_{\alpha j} = \rho u_i u_j + p\delta_{ij}, \tag{3c}$$

$$\sum_\alpha f_\alpha^{eq} e_{\alpha i} e_{\alpha j} e_{\alpha k} = \rho u_i u_j u_k + p\left(u_k \delta_{ij} + u_j \delta_{ik} + u_i \delta_{jk}\right), \tag{3d}$$

$$\sum_\alpha f_\alpha^{eq} e_\alpha^2 e_{\alpha i} e_{\alpha j} = \rho u^2 u_i u_j + p\left[(D+2)RT\delta_{ij} + (D+4)u_i u_j + u^2 \delta_{ij}\right], \tag{3e}$$

where $\rho$ is the density and $p = \rho RT$ is the pressure, in which $R$ and $T$ are the specific gas constant and the temperature. $D$ is the dimension of the space, and the subscripts $i$, $j$, and $k$ indicate the $x$, $y$, or $z$ components, respectively. $\delta_{ij}$, $\delta_{ik}$, and $\delta_{jk}$ are the Kronecker delta functions. It should be noted that $f_\alpha^{eq}$ in Li et al.'s model [26] did not satisfy Eq. (3e). Since, in their model, $f_\alpha^{eq}$ is in charge of the mass and momentum equations, while $h_\alpha^{eq}$ is for the energy equation. Nevertheless, according to the non-equilibrium statistical physics, the local flow density, momentum and energy should be described by the same distribution function. To reduce the potential inconsistence of this model, we introduce D2Q17 circle function to replace D2Q13 circle function for first to fourth order constraints of $f_\alpha^{eq}$. Thus, the heat flux and the energy dynamics of $f_\alpha^{eq}$ are also accurate up to the Navier–Stokes level. In addition, $h_\alpha^{eq}$ should satisfy the following conditions:

$$\sum_\alpha h_\alpha^{eq} = \rho E, \tag{4a}$$

$$\sum_\alpha e_{\alpha i} h_\alpha^{eq} = (\rho E + p) u_i, \tag{4b}$$

$$\sum_\alpha e_{\alpha i} e_{\alpha j} h_\alpha^{eq} = (\rho E + 2p) u_i u_i + p(E + RT)\delta_{ij}, \tag{4c}$$

where $E = bRT/2 + u^2/2$, $b$ is a constant, which has the relationship with the specific-heat ratio $\gamma$ as $\gamma = (b+2)/b$.

### 2.2. Discrete velocity model

Li et al. introduced D2Q13 circular function proposed by Qu et al. into the coupled DDF-LB model for flows with high Mach number [26]. In fact, $f_\alpha^{eq}$ based on D2Q13 circular function can