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Numerical simulation of the temporal evolution of a three dimensional barchanoid dune and the corresponding sediment dynamics

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ABSTRACT

In this paper we present the results of the numerical simulation of a three-dimensional current-driven sediment transport process. In detail, the temporal evolution of a barchanoid dune is studied. Two phenomena are treated in this context. First, the three-dimensional flow of a single phase fluid is considered. Second, the interaction of the flow and the sediment bed with its morphological change of the sediment surface is taken into account. We numerically solve the instationary incompressible Navier–Stokes equations, an advection diffusion equation and Exner's bed level equation to update the sediment bed morphology. Here, Exner's equation determines the change of the bed level due to the bed load. The suspended material is treated as a sediment concentration and its movement is modelled by an advection-diffusion equation. To secure the continuous interchange between bed load and the suspension load sink and source terms are used. Both equations are discretised and explicitly coupled to the discrete fluid model. The typical sedimentary processes and the sedimentary form of a prototypical barchanoid dune are well captured by our numerical simulation, which is supported by a qualitative comparison with examples from the literature.

1. Introduction

Sediment transport processes and their effects on the morphology of the sediment bed are significant issues in hydraulic engineering. Usually, the physical processes of the formation of dunes and other sedimentary forms are studied in laboratory flumes or in field experiments. These time-intensive and costly studies are not always easy to conduct. At this point, a numerical simulation can help to reduce costs and to provide more insight and therefore a better understanding of the relevant flow and transport phenomena.

There are different classifications of dunes in the aeolian regime as well as in the fluvial regime. For example, linear dunes, crescent shaped dunes, e.g. parabolic or barchanoid dunes, and star shaped dunes demonstrate the large diversity of dune forms. Here, the availability of sand, its consistency, the predominant wind situation and many other factors determine the dune type, compare [24]. In general, the sediment is transported in the bed load layer over the dune body upwards the upstream slope. When the sand particles are transported to the top end of the dune, the particles slide down the downstream slope, which is limited by the angle of repose. In case of a barchanoid dune, the transport velocities are higher near the lateral ends of the dune body. This fact leads to a faster transport of the sand at the sides of the dune body and to the development of sand horns, which are transported further downstream. The resulting dune body and the involved processes are strictly three-dimensional. We present a numerical approach for their simulation and discuss the obtained results.

The remainder of this paper is organised as follows. In Section 2, we describe the full fluid-sediment-model, which consists of the Navier–Stokes equations, a suspension load model, and Exner's bed level equation. In Section 3, we shortly discuss our numerical discretisation and its properties. In Section 4, we present the results of our numerical simulation for the temporal evolution of a barchanoid dune. A conclusion is given in the fifth section.

2. Model: Navier–Stokes, sediment transport and surface model

The used model comprises a three-dimensional fluid model and the sediment equations, which realise the suspension load transport and the morphological change of the sediment surface. Parts of the presented models were previously studied in the literature, e.g. [17,23,33–35,41]. Some authors already combined a two or three dimensional fluid solver with a sediment model for the morphological change ([4,34,35]) or the suspension load [5,41].

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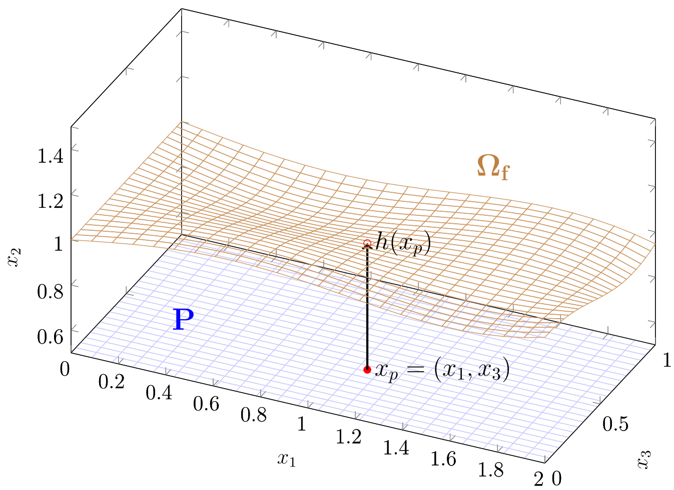

Fig. 1. The sediment surface is described for each time point t by its height $h(x_p)$ with $x_p = (x_1, x_3)$, i.e. the distance from an underlying plane $P = (x_1, x_3)$. Thus, the fluid domain Ω_f is bounded by $h(x_1, x_3)$ from below.

In this chapter, we introduce a full three dimensional loosely coupled algorithm for all three models.

2.1. Navier–Stokes equations

Due to the complex three-dimensional character of sedimentary bedforms and especially dunes, it is necessary to apply a full three-dimensional model. Here, the instationary incompressible Navier–Stokes equations in their dimensionless form read as

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u} \otimes \mathbf{u}) = \frac{1}{Fr^2}\mathbf{g} - \nabla p + \frac{1}{Re}\Delta \mathbf{u}, \tag{1a}$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{on} \quad \Omega_f \in \mathbb{R}^3, \tag{1b}$$

where $\mathbf{u}$ is the velocity, p is the pressure, $\mathbf{g}$ are the volume forces, and Ω_f denotes the domain of the fluid body. Moreover

$$Re = \frac{\mathbf{u}_\infty \cdot l}{\nu} \tag{2}$$

denotes the Reynolds number and

$$Fr = \frac{\mathbf{u}_\infty}{\sqrt{g \cdot l}} \tag{3}$$

denotes the Froude number. Both, Re as well as Fr, are dimensionless numbers which characterise the flow conditions. The characteristic length and velocity are denoted by l and $\mathbf{u}_\infty$, respectively. As usual, ν stands for the kinematic viscosity of the fluid. As boundary conditions no-slip Dirichlet boundary conditions

$$\mathbf{u} = 0 \tag{4}$$

and slip Neumann boundaries conditions

$$\frac{\partial \mathbf{u}}{\partial n} = 0 \tag{5}$$

are applicable at the boundary of the fluid domain Γ_{Ω_f}.

2.2. Sediment surface and the Exner's bed level equation

The Navier–Stokes equations are solved on a time-dependent fluid domain Ω_f whereas the bottom of this domain is bounded for each time point by its sediment surface $h(x_1, x_3)$. This sediment surface h describes the height of the underlying sediment with respect to a reference plane $P = (x_1, x_3)$, compare Fig. 1. To model the temporal change of the sediment surface h, we use the bed level equation postulated by Exner [19], i.e.

$$\frac{\partial h}{\partial t} + \nabla_{(x_1, x_3)} \cdot \mathbf{q}_s(\tau(\mathbf{u})) = 0 \qquad \text{on } P, \tag{6a}$$

$$\frac{\partial h}{\partial n} = 0 \qquad \text{on } \Gamma_P, \tag{6b}$$

where $\mathbf{q}_s(\boldsymbol{\tau}(\mathbf{u}))$ is the transport rate function of the sediment and the gradient operator with respect to (x_1, x_3) is denoted by $\nabla_{(x_1,x_3)}$. The transport functions $\mathbf{q}_s$ depends on the shear stress τ, which is a function of the fluid velocity $\mathbf{u}$, where $\tau(\mathbf{u})$ is here just needed on the sediment surface. In the Neumann boundary condition (6b) the normal is denoted by n. The Exner equation states that the net balance between gain and loss of mass in a certain control volume results in a change of the sediment height h. It was successfully used in several studies to investigate the evolution of geomorphological change, e.g. [34,35,43,44]. The presented model results from the conservation of mass and therefore from first principles. Moreover, Coleman and Nikora [10] used a statistical averaging process of a granular bed over time and space to derive the Exner equation.

The sediment surface determined by h denotes implicitly the fluid domain Ω_f. Thus, a change in h results in a change of the fluid domain Ω_f. Several models for the shear stress $\tau : \mathbb{R}^3 \mapsto \mathbb{R}^2$ and the transport rate function $\mathbf{q}_s : \mathbb{R}^2 \mapsto \mathbb{R}^2$ are available in the literature, see [6]. In the following, we choose the empirically derived models (7a) and (7b)

$$\mathbf{q}_s = \varepsilon\sqrt{(s-1)gd_{50}^3} \cdot \left(\frac{4\tau(\mathbf{u})}{\rho_f(s-1)gd_{50}} - \tau^*\right)^{\frac{3}{2}}, \tag{7a}$$

$$\tau(\mathbf{u}) = \frac{1}{8}\rho_f f |\mathbf{u}|^2, \tag{7b}$$

where ρ_s denotes the sediment density, d_{50} is the median grain size, τ^* is the dimensionless critical shear stress and $s = \rho_s/\rho_f$ with ρ_f being the fluid density. Note, that the bed load includes a porosity constant ε which describes the density of the packing of the grains. The friction parameter in (7b) is set according to Chanson [6] as

$$f = \frac{64}{Re}. \tag{8}$$

Chanson [6] proposed formula (7b) as a modified version of the transport formula from [39], which has been validated by numerous experimental studies. Wong and Parker [53] gave a nice summary and analysis of (7a). Note, that we do not apply any slope correction in the bed load calculation as proposed in the literature, compare [8]. Later, in Section 2.6 we use a slope limiting algorithm which distributes the masses according to the slope.

2.3. Suspension load model

The suspension load comprises all material which is transported in the whole fluid. An advection-diffusion model is used to describe the transport of the suspended material.

Since only very fine grains are transported, the common approach is to model the entrained material as a concentration c of mass in the fluid domain. Similar to the bed level equation, the advection-diffusion model for the suspended material can be derived from the conservation of mass and momentum. Malcherek [36] presented a suspension model as

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c + w_g \frac{\partial c}{\partial x_2} = K\Delta c \qquad c(\mathbf{x}, t) : \Omega_f \times [0:T] \longrightarrow \mathbb{R}, \tag{9a}$$